JIPM (Jurnal Ilmiah Pendidikan Matematika) 12(2), 2024, 310-320
DOI : 10.25273/jipm.v12i2.18833

JIPM (Jurnal Ilmiah Pendidikan Matematika)
Journal homepage: http://e-journal.unipma.ac.id/index.php/jipm

Students' Ways of Understanding and Ways of Thinking in Solving Trigonometric Problems

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 ISSN 2337-9049 (print), ISSN 2502-4671 (online)

Abstrak: Berpikir matematis sangat penting sebagai cara untuk mempelajari matematika. Menurut teori Harel istilah berpikir matematis tingkat lanjut berakar pada dua elemen pengetahuan: cara memahami (WoU) dan cara berpikir (WoT). Cara memahami mengacu pada bagaimana siswa menginterpretasikan suatu konsep tertentu dan hubungan antar konsep, memberikan solusi terhadap masalah, dan membenarkan apakah suatu keyakinan benar atau tidak, sedangkan cara berpikir adalah proses mental yang menghasilkan pemahaman siswa terhadap suatu hal. Tujuan dari penelitian ini adalah untuk menggambarkan cara siswa memahami dan berpikir dalam menyelesaikan masalah matematika yang terkait dengan trigonometri. Penelitian ini dilakukan di sebuah sekolah menengah atas di Jakarta Selatan, Indonesia. Subjek penelitian adalah siswa kelas 11 yang telah mempelajari tentang trigonometri. Data dikumpulkan dari instrumen tes yang terdiri dari tiga masalah matematika terkait trigonometri dan hasil wawancara. Analisis data dilakukan dengan tiga tahap, yaitu teknik reduksi, penyajian, dan penarikan simpulan. Hasil penelitian ini menunjukkan bahwa cara memahami dan cara berpikir siswa saling memengaruhi. Jadi, dalam masalah trigonometri ini, cara berpikir siswa memengaruhi pemahaman siswa terhadap konsep, dan sebaliknya pemahaman siswa terhadap konsep memengaruhi cara berpikir mereka.
Kata kunci: *Cara berpikir; Cara memahami; Trigonometri; Studi Kasus; Penelitian Kualitatif.*

Abstract: *Mathematical thinking is essential as a way of learning mathematics. According to Harel's theory in defining the term advanced mathematical thinking, it is rooted in two pieces of knowledge: ways of understanding (WoU) and ways of thinking (WoT). Ways of understanding refer to how students interpret a particular concept and the relationship between concepts, give solutions to a problem, and justify whether a belief is true or not, while ways of thinking are a mental process that results in students' understanding of a particular thing. The purpose of this research is to describe students' ways of understanding and ways of thinking in solving mathematical problems related to trigonometry. The research was conducted at a senior high school in Jakarta Selatan, Indonesia. The subjects of the research were the students grade 11 who has learned trigonometry. The data were collected from the test instruments consisting of three mathematical problems related to trigonometry and interview transcript. The data analysis used in this research are reduction techniques, presentation, and drawing conclusions. The result of this research showed that students' ways of understanding and ways of thinking influenced each other. So, in this trigonometry problem, students' ways of thinking influence the students' understanding of the concepts, and likewise students' understanding of the concepts affects their ways of thinking.*
Keywords*: Ways of thinking; Ways of understanding; Trigonometry; Case study; Qualitative research.*

Introduction

Mathematics is a branch of science that is very much needed because of its various benefits for everyday life. According to the Minister of National Education Regulations (Permendiknas) No. 22 one of the purposes of learning mathematics is that students are able to understand the concept of mathematics, establish reasoning ability, problem-solving, and have an attitude of appreciating the usefulness of mathematics in life (BNSP, 2016). Mathematics is one of the ways to construct students' thinking skills Hendriana and Soemarmo (2017) stated that there are four aspects of how mathematics is taught in learning, namely presentation, mindset, universal conversation, and level of abstraction that must be adapted to students learning development. This leads to a question about whether the learning process at school has accommodated students to enhance their thinking skills because it is clear that the purpose of learning according to the national regulation is to develop students thinking skills.

Thinking is a mental activity that every person does in their daily life. By thinking, someone knows the purpose of what he/she does. When someone thinks, he/she start from something that is called by assumptions then this leads to the end of the thinking where the implication and consequences of someone's logic show up as the result of thinking (Arisoy & Aybek, 2021). Thinking also allows someone to get ideas or thoughts about what they are thinking (Sunaryo, 2011). Every student is able to think because they generally have mathematical abilities in ways of thinking and understanding since their early stage of childhood (Nirawati et al., 2022). For example, critical thinking which is an important tool that students should use to solve the problems they face in their daily life because it involves logical reasoning, interpreting, analyzing, and evaluating information to enable them to make reliable and valid decision (Widana, 2018). Due to the 21st century skills, students need to develop the ability to understand the concepts, improve high-quality thinking skills in analyzing concepts, evaluating, and finding alternative solutions to solve problem (Jayanti et al., 2018). Students who have a good thinking skill they will be able to solve problem fluently because they not only understand the concept, theorems, or definitions but also, they can connect everything they have learnt to solve problem or make conclusion. One kind of thinking skills that is required for students is mathematical thinking.

Sometimes, we are confused about what it is called by mathematical thinking, is it a kind of thinking about mathematics, or another type of thinking skills using mathematics. Mathematical thinking is a process when students' mind systematically performs their ability to find solution of mathematical problems which lead them to think, to review, and to arrange their previous mathematical experiences to reach the final solution (Jawad et al., 2021). Another opinion about mathematical thinking when it comes to students mathematical thinking, it is about the development of important mathematical ideas (Leatham et al., 2015). In addition, it is the cornerstone of development of mathematics because it helps someone to visualize the abstract mathematical relations in the mathematical applications to reach the highest levels of abstractions (AlAli et al., 2023). On the other hand, Burton stated that mathematical thinking is different from thinking about the subject of mathematics. It is specifically a style of thinking related to the four processes, specializing, conjecturing, generalizing, and convincing (Burton, 1984). This is in line to what Devlin mentioned that mathematical thinking is about mastery of a particular way of thinking (Devlin, 2021). Therefore, mathematical thinking is not only can be applied in learning mathematics itself but also can be applied to solve any problem such as the used of scientific method. Way of thinking is part of knowledge that will help us in defining mathematical thinking or advanced mathematical thinking. Harel's theory asserts that advanced mathematical thinking is rooted on two pieces of knowledge: Ways of Understanding denoted by WoU, and Ways of Thinking denoted by WoT (Harel & Sowder, 2005). Ways of

understanding and ways of thinking are clearly different, yet it is related and influence each other.

Ways of understanding refer to the product such as definition, conjectures, theorems, proofs, problems, and solutions, whereas ways of thinking refer to the act that resulted those products (Harel, 2008b, 2008a; Harel & Koichu, 2010; Harel & Sowder, 2005). Ways of understanding and ways of thinking are called as duality means a reciprocal relationship as a part of the theoretical framework and principles of DNR-based instruction, which then short DNR; duality, necessity, and repeated reasoning (Bakar et al., 2019). It is also called by a reciprocal process to develop students' ways of thinking because one can't be done without the development of students' way of understanding. This process happens when students are at the center of learning and building their knowledge that leads to the development of students' learning skill which focuses on the developing mindsets and ways of understanding the mental actions of the students (Nirawati et al., 2021). In other word, the two terms ways of understanding and ways of thinking not only influence each other but also the changes in ways of understanding will not occur without the changes in ways of thinking. This indicates that to learn mathematics, students need to build a good way of understanding and way of thinking.

The role of teacher in teaching mathematics takes an important part as building students' way of understanding and way of thinking. It involves the analysis of students' learning obstacle and suitable learning trajectory. In the context of working on solving problem, it requires systematic and logical activities where students are expected to understand the problem, to identify what is known and what is being asked, then they can easily use mathematics procedures to solve it. In the process of solving problem, students will conduct a mental act which is known as ways of understanding such as interpreting the problem, determine the concepts that should be used to solve problem, do the calculation, then evaluate or validate whether the answer is correct.

Trigonometry is one of the materials that should be learned by students in school. It is related to angles, borders of triangles, and commonly used to measure length (Nanmumpuni & Retnawati, 2021). Trigonometry is a new thing for students in senior high school, that is why the basic concept of it had to be learned meaningfully so they can understand trigonometry well (Faizah et al., 2021). Students understanding in trigonometry is substantial because it is related to students' reasoning ability and also critical to develop mathematical understanding (Eko et al., 2018). However, memorization of the formula is the most common way students tend to do to solve trigonometric problem (Rohimah & Prabawanto, 2020). In some cases of solving problem, students misinterpret the problem and having difficulties in getting the essential information from the problem (Rizky Novriani & Surya, 2017) even though understanding the problem is half way to solve it. In line with this, the previous study from Maknum (2022) reveals that there are epistemological obstacles in learning trigonometry, which lead to the students' understanding of the trigonometry and trigonometry functions. Another finding from Nurmeidina denoted that in solving problem related to trigonometry, students are having lack of understanding in determine procedures to execute the problems and formulating mathematical sentences (Nurmeidina & Rafidiyah, 2019). Based on the existing findings, there are problems in learning trigonometry which make students feel that trigonometry is difficult to be learned. In addition, the studies about students' misconception in trigonometry, students' errors in solving trigonometric problems, and students' difficulty in solving trigonometric problems has been done recently, however the study that focuses on students' mathematical thinking especially study about students' ways of understanding and ways of thinking in solving trigonometric problem has not been done. In this study, there are three problems that will be given to the students and the problems are used to identify students' ways of understanding and ways of thinking in solving trigonometry.

According to Harel, that ways of understanding and ways of thinking are part of mathematical thinking. Thus, it is important to know how is students' mathematical thinking so that teachers can provide sufficient assistance in improving them. This research will focus on how the students' ways of understanding and ways of thinking in solving problem related to trigonometry. Therefore, this study aims to explore and interpret students' ways of understanding and ways of thinking in solving trigonometric problems.

Method

This type of study was qualitative descriptive research. This study used descriptive methods to explore and interpret students' ways of understanding and ways of thinking in solving trigonometry problem. The research subject was chosen purposively. There were 3 students who had studied trigonometry with different ability in mathematics. They were chosen from one of the Senior High School in Jakarta Selatan.

The data collection consisted of test instrument and interview. There were three questions in the test instrument. First, students worked on the problems then conduct the interview to deepen students' experience in solving the problem and to explore their ways of thinking and ways of understanding when working on the problems. The result from the test instruments were the primary source in assessing students' ways of thinking and ways of understanding. The data analysis used in this research were data reduction, data display, and conclusion drawing/verification. Data reduction was done to collect the data that would be used and relevant to the research, then data display was conducted to show the result of the calculated data, thus it helped the researcher to work on the conclusion and decision-making, and the last was conclusion drawing where the researcher determined the meaning of the data that had been collected and analyzed.

This research adapted the characteristic of ways of understanding and ways of Thinking from Nurhasanah as followed in the Table 1. (Nurhasanah & Jupri, 2021):

Table 1. Characteristics WoU and WoT

Characteristics	Ways of Thinking		
	Category	Right (R)	Wrong (W)
Ways of Understanding	Right (R)	RR	RW
	Wrong (W)	WR	WW

This table will lead the researcher to focus on four characteristics to be studied, namely: the right way of understanding and the right way of thinking (RR), the right way of understanding and the wrong way of thinking (RW), the wrong way of understanding and the right way of thinking (WR), and the wrong way of understanding and the wrong way of thinking (WW).

Results and Discussions

According to the characteristic of ways of understanding and ways of thinking, each of characteristics has different properties. These properties were adapted from Nurhasanah research results which were adjusted to the material from the research being studied as followed in the Table 2:

Table 2. Properties of WoU and WoT

Characteristics	Descriptions
RR	Students are able to use concepts, solve the problem systematically, and understand which trigonometry used (sine, cosine, or tangent) to solve the problem. Also, they can identify what is being asked and find the best way to figure out the solution.
RW	Students understand which concept and which trigonometry (sine, cosine, or tangent) is used according to what the type of triangle, yet students struggling how to work on the strategy or steps to find the solution of the problem systematically.
WR	Students do not understand what the concept used to solve the problems and struggling in identify the information needed to solve the problem, yet they can describe the steps of how to do the calculation and shows ways of thinking correctly and logic.
WW	Students do not understand the concept of trigonometry in a triangle and find it difficult to conduct the strategy to solve the problem systematically.

To gain the more specific explanation about how students' ways of understanding and ways of thinking according to the specific properties of the four characteristics found. The Author has divided the information through some points below.

Right ways of understanding and right ways of thinking

In problem 1, it is stated that the standard slope for making ramp with a height of 1 m is 2.86^{o}, students were asked to find the length of the ramp path or the length of the slope from the ideal standard that have been determined.

Problem 1.
The standard slope for making ramps as facilities for disabled people, children, and elderly people with a height of 1 m is 2.86^{0}. what is the length of the ramp path that is formed from the ideal standards that have been determined. ($\sin 2.86° \approx 0.0\,498, \cos 2.86° \approx 0.9987$)

Figure 1. Question 1

Students seemed that they have no difficulties in solving this problem. Students could use their reasoning to carried out the concepts, solve the problem, and understand which trigonometry used accurately. Students used sine instead of cosine to find the length of the ramp which was here as the hypotenuse.

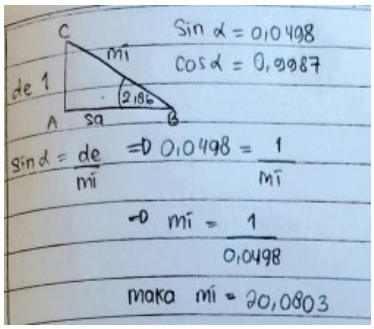


Figure 2. Student's answer in the category of RR

From the Figure 1, it was obvious that students' ways of understanding and ways of thinking were right. This indicated that students the students had good reasoning ability so they could apply the best strategy to solve the problem (Sari et al., 2023). Moreover, they illustrated the ramp and identify each of the side before working on the strategy and calculation to solve the problem. This result was supported by the interview as followed:

Researcher: How did you solve the problem 1?

Student 1: I drew the triangle, because it was a right triangle, then I can find the length of path ramp using the ratio of trigonometry of sine.

Researcher: Why did you use sine instead of cosine?

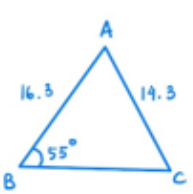
Student 1: Because, it depended on the information given in the problem. It was easier for me to use sine rather than cosine.

Right ways of understanding and wrong ways of thinking

In problem 3, students were required to evaluate whether the answer of both students were correct, wrong or only one of it was correct. Ahmad and Bayu gave the different answer to the problem. They draw their triangle according to what they understand and thinking of how the problem will be best solved.

Problem 3.

In Δ ABC, $\angle ABC = 55°$, $AB = 16.3\ cm$, dan $AC = 14.3\ cm$. Ahmad dan Bayu obtained different value of $\angle ACB$.



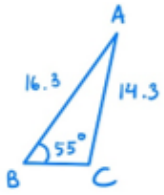
Ahmad's answer

$$\frac{\sin \angle ACB}{16.3} = \frac{\sin 55°}{14.3}$$

$$\sin \angle ACB = 0.93372$$

$$\angle ACB = \sin^{-1} 0.93372$$

$$\angle ACB = 69°$$



Bayu's answer

$$\frac{\sin \angle ACB}{16.3} = \frac{\sin 55°}{14.3}$$

$$\sin \angle ACB = 0.93372$$

$$\angle ACB = 180° - \sin^{-1} 0.93372$$

$$\angle ACB = 111°$$

Figure 3. Problem 3

Figure 4 showed that students tried to evaluate Ahmad's answer by calculating the value of $\angle ACB$ from Ahmad's calculation. Resulted that the $\angle ACB$ was $69°$, then student conclude that Ahmad's answer was correct.

After students figured out that Ahmad's answer was correct, student did not try to evaluate Bayu answer. It happened because they speculate that only one answer was correct and the other was wrong. Moreover, they speculate the weirdness from Bayu's answer especially in the part of $180° - sin^{-1}\,0.93372$. They thought that it was unnecessary to subtract the angle from $180°$. They also did not notice that the triangles were drawn by Ahmad and Bayu were different. One had acute angles whereas other had an obtuse angle.

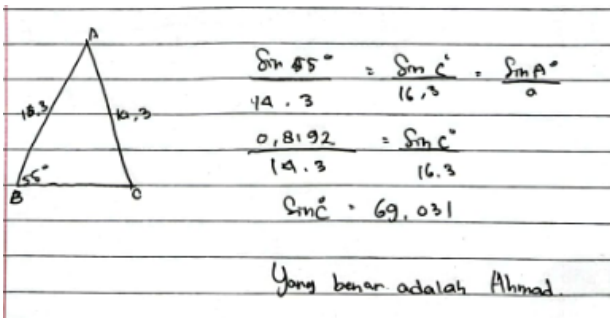


Figure 4. Student's answer in the category of RW

In this Figure, the answer showed that students still struggling to build their ways of thinking in the problem that required evaluation check. This result was in line with the interview conducted with the student as followed:

Researcher: How did you solve the problem 3?

Student 2 : I calculated Ahmad's answer by myself and found that the answer was fit with mine.

Researcher: If Ahmad answer was correct, so do you think that Bayu's answer is incorrect?

Student 2: Yes, because Bayu's answer was gotten from subtracting the angle from $180°$, that actually was unnecessary.

Researcher: Did you notice that Ahmad's and Bayu's triangle were different?

Students 2: Yes.

Researcher: Do you think was it possible to have all the answer correct?

Students 2: No, not really. Because I think Bayu's answer was weird and incorrect, so I only check Ahmad's answer.

This result also described that students tended to validate their answer by searching mathematical correctness and leave the searching for sensibility behind. According to Prabawanto that sensibility refers to the condition of building reassurance whereas the idea is reasonable (Prabawanto, 2019). This was the reason of why students only evaluate from one perspective answer and used the result to make generalization.

Wrong ways of understanding and right ways of thinking

In problem 2, it is stated that a ship sailed eastward for 60 km then changed the direction by turning the rudder as far as $60°$ until it stopped. Students need to find the distance between the starting point to the end point where the ship stopped. To solve this problem, students are required to sketch the information given, so they can have a clear picture about the position of the ship from the starting point to the end point. This problem is the application of cosine rule and also related to bearings.

Problem 2.
The ship sailed eastward for 60 km, then turned the rudder in a direction of 60° for 50 km until it stopped. The distance of the ship from the starting point of sailing to the stopping point is … m.

Figure 5. Problem 2

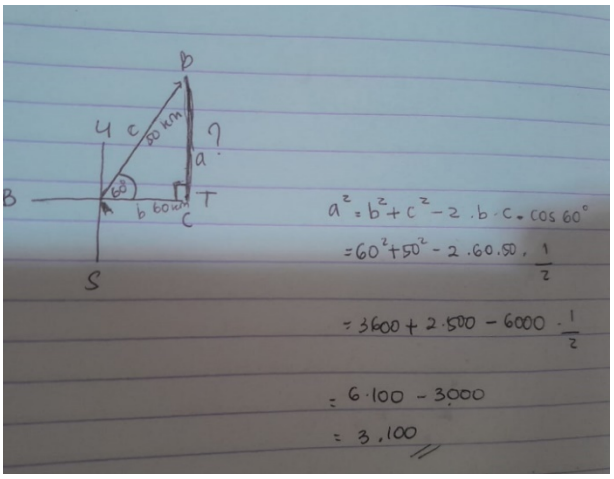


Figure 6. Student's answer in the category of WR

This answer showed that students did not draw the path where the ship sailed accurately. Student drew the angle right at the starting point A then draw a line as the path where the ship sailed to the point B where the ship stopped. Afterward, student connected the starting point A to the end point B by projecting both points then make a right triangle. This drawing led student to see a right triangle which actually was not.

This implied that student's ways of understanding about starting point, bearing, and distance between the starting point to the end point needed to be developed. Even students used the correct strategy, here was the formula and the prejudice that the triangle was not the right triangle. Their lack of understanding of the information given in the problem 2 and concept of bearing led them to draw the path incorrectly.

This result revealed that students' ways of understanding have a significant impact to their ways of thinking, and it possible to be the vice versa. This was also confirmed by interview with Student 3 as followed:

Researcher: How did you solve the problem 2?
Student 3: I thought it would be a triangle, then I drew the path where the ship sailed and found out it was a right triangle.
Researcher: Why did you think it was a right triangle?
Student 3: I don't know, but when I drew, it came out a right triangle.
Researcher: How did you draw the angle when the rudder of the ship turned 60°?
Student 3: I drew it from the horizontal line counterclockwise, then I create a line and label it 50 km as the information given.
Researcher: You put the angle of 60° at the starting point A. Did you draw it there because you used to learn the four quadrant of trigonometry angles?
Student 3: Yes, I did.

This condition however is in line with the previous study, which explained that trigonometry is difficult and confusing for some students due to the graphics, algebra, geometry and any other disciplines that related to it (Marufi et al., 2022). When working on the problem, students are required to be able to represent the information through the graphic form, so it is not enough to just knowing the formula used to solve the problem. In

fact, only a few students involved reasoning in solving trigonometry problems, such as how they claim the graph they made, whether it is correct or not, it represents the information or not, and it can be proven or not. Therefore, it is not surprising if students misunderstand the problem and this misunderstanding leads them to make mistakes in executing the solution of the problem presented.

Wrong ways of understanding and wrong ways of thinking

This was another answer from student's work in solving the problem 2. The Figure 4 showed that student did not understand the concept of bearing. It could be seen from the angle which student has drawn. The angle was drawn from the horizontal line not the vertical line that indicated as North. Although the direction was clockwise, it still could not be right because the angle had to be drawn from the North as $0°$.

This answer also showed that student used the different approach to find the distance between the starting point to the stopped point. The student used the ratio of trigonometry for the right triangle which led to the way harder to solve because of the limited information that provided in this problem. This indicated that students find it difficult to connect all the information provided to find the easier way by applying the cosine rule. Therefore, this answer was categorized as WW.

Every character of ways of understanding and ways of thinking had an important role in students' mental act. Students' ways of thinking influence students' ways of understanding, and so is the students' ways of understanding influence students' ways of thinking (Nurhasanah & Jupri, 2021). Therefore, it is important to know the level of students' ways of understanding and ways of thinking in order to develop a good mathematical thinking.

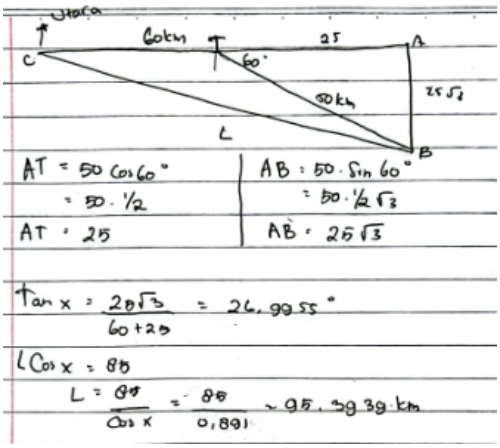


Figure 7. Student's answer in the category of WW

Conclusion

Based on the analysis of students' work in solving the trigonometry problems, students' ways of understanding and students' ways of thinking influence each other. Students who understand the concept trigonometry will not easily able to solve the problem without good ways of thinking and so do the students who has good ways of understanding will not able to solve problem without having good ways of understanding. The main things that affect students' ways of understanding and ways of thinking are the students' initial concept mastery and the learning obstacle which students have.

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