

MEASUREMENT OF INTEREST RATE AND PERIOD USING LOGARITHM EQUATIONS: A STUDY OF TIME VALUE OF MONEY

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ABSTRACT

The primary aim of this study is to illustrate how logarithmic calculation and equation can effectively replace traditional table usage and interpolation techniques in time value of money calculations. It proposes the application of algorithmic computation to calculate the interest rate and period when the future value and present value are known. It developed and formulated two primary equations utilizing an algorithmic approach as an alternative to the conventional reliance on financial tables and linear interpolation prevalent in the existing literature. The two methods are validated and substantiated upon reevaluating the fundamental time value of money calculation, yielding far more precise results than the traditional method. The measurement can also be utilized in the statistical realm to compute the compounded average yearly growth or growth period when other parameters are available.

Keywords: Algorithm; Linear interpolation; Interest rate; Period; Time value of money

ABSTRAK

Tujuan utama dari studi ini adalah untuk memberikan ilustrasi bagaimana perhitungan dan persamaan logaritma dapat secara efektif menggantikan penggunaan tabel tradisional dan teknik interpolasi dalam perhitungan nilai waktu uang. Studi ini mengusulkan penerapan perhitungan algoritma untuk menghitung suku bunga dan periode waktu ketika nilai masa depan dan nilai kini diketahui. Studi ini mengembangkan dan merumuskan dua rumus utama yang memanfaatkan pendekatan algoritma sebagai alternatif dari ketergantungan konvensional pada tabel keuangan dan interpolasi linier yang lazim dalam literatur yang tersedia. Kedua metode tersebut tervalidasi dan setelah mengevaluasi ulang perhitungan nilai waktu uang ke rumus dasarnya, menghasilkan hasil perhitungan yang lebih akurat daripada metode tradisional. Pengukuran tersebut juga dapat digunakan pada bidang statistik untuk menghitung pertumbuhan tahunan rata-rata atau periode pertumbuhan ketika parameter lain tersedia.

Kata Kunci : Algoritma; Interpolasi linear; Tingkat bunga; Periode, Nilai waktu uang

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INTRODUCTION

Financial managers, investors, and other business professionals are constantly looking for new ways to generate a good return on their assets (Jeffers et al., 2024; Pástor et al., 2022; Kanuri, 2020), whether through investments in attractive projects, securities with an inevitable return, or bank deposits. In this scenario, the rate of return on investment and other factors are closely tied to the time value of money (Narayan et al., 2020; Drechsler et al., 2021; Lind et al., 2013). The theoretical foundation of the time value of money (TVM) is essential in financial and accounting measures and computations (Brealey et al., 2023; Garrison et al., 2021; Baehaqi et al., 2020). The concept is predicated on the belief that one dollar now is worth something different than a dollar tomorrow (Muniesa & Doganova, 2020; Dai et al., 2022; Weber, 2021).

In revisiting the fundamental equation of the time value of money, four components are considered in the calculation: (1) Future Value (FV), representing the value of a specific cash flow at a later time; (2) Present Value (PV), denoting the current value; (3) Interest Rate (i), the assumed rate of interest; and (4) Period (n), indicating the number of periods involved in the calculation. Accounting and financial management literature has formulated equations to determine the initial two parameters from the aforementioned four components (i.e., Busu, 2022; Slobodnyak & Sidorov, 2022; Drake & Fabozzi, 2009). Furthermore, different computational tools have been devised to facilitate computations, including tables, financial calculators, and computer spreadsheets (Campbell & Brown, 2022; Brigham & Houston, 2019; Cecchetti & Schoenholtz, 2017). Nevertheless, for the latter two components, despite the prevalent utilization of diverse calculating tools such as tables, financial calculators, and computer spreadsheets, the fundamental method for determining the interest rate and period from known present and future values is not well developed.

Time value of money (TVM) is a fundamental component of business and finance education (Newfeld, 2012; Chen, 2009). However, according to Jalbert and Stewart (2023), the concept of time value of money (TVM) has proven difficult for students to grasp and for teachers to communicate. Many financial calculations have traditionally relied on tables and interpolation methods to determine interest rates and periods (i.e., Biegel, 2022; Cotter, 2021; Kieso et al., 2019). These approaches can be cumbersome for students and may lead to inaccuracies, especially in complex scenarios. According to Dempsey (2003), while accounting and finance both rely heavily on the time value of money, the topic is among the most challenging for undergraduate accounting and finance students. He argued that many students' knowledge still needs to improve even after a third exposure to an upper-level curriculum that heavily uses discounting principles. Furthermore, he adds that considering that solutions manuals for accounting and finance texts seldom present answers using mathematical formulations, either exclusively or in conjunction with table references, it is reasonable to infer that the predominant emphasis is on tables. Consequently, although financial calculator features and spreadsheet software are increasingly utilized in the TVM calculation (Alexander, 2021; Badiru & Mertens, 2023; Jalbert & Jalbert, 2019), it is essential to establish a robust understanding of the mathematical foundations of TVM among users.

The study proposes an alternative method using a mathematical logarithmic calculation and equation to measure interest rates and periods. Utilizing logarithms can eliminate the need for extensive tables and interpolation methods, thereby streamlining the calculation process. Logarithmic equations simplify the mathematical operations involved and enhance the accuracy of the results, making them more

accessible to accounting and finance professionals, students, and laypersons. This article is organized along three objectives. First, using an algorithm approach, it provides mathematical equations for calculating interest rates from a single payment of time value of money. Second, using a similar algorithm approach, it provides mathematical equations for calculating periods when other parameters are known. Third, it explains how these equations can also be utilized in statistical calculations to measure growth with exact results.

METHOD

This study aims to thoroughly examine how logarithms facilitate complex financial calculations and enhance the precision of financial modeling. The approach incorporates theoretical frameworks with practical data analysis to address the study objectives thoroughly. The initial stage in establishing the theoretical framework for the inquiry is to analyze the conventional model. This study analyzes the interest rates with particular emphasis on traditional methods and their limitation. It also provides a solution for knowledge gaps by integrating existing studies and recommending using logarithmic methods to ease the computation of interest rates and periods.

The research continues to create mathematical models that apply logarithmic equations to calculate interest rates and periods. This stage entails creating equations that represent interest calculations in logarithmic terms, such as the relationship between present value (PV), future value (FV), interest rate (r), and time (t), using the following equation (Brealy et al., 2023; Dalquist & Knight, 2022; Kieso et al., 2019).

$$FV = PV + (1+k)^n \dots\dots\dots(Equation 1)$$

Description:

- FV = Future value
- PV = Present value
- k = interest rate
- n = period

The next step involves rearranging the equation using the logarithm model to derive the interest rate and period equations. The steps include generating two logarithmic equations to calculate the interest rate and period based on known present and future values. The step is followed by testing the proposed equation, verified by numerical examples showing how logarithmic functions can be used in different financial situations. Numerical examples highlighting the accuracy and computing efficiency of logarithmic computations compared to more conventional approaches will be used to validate the models. This step is essential for creating a theoretical framework that backs up the established models' actual implementation.

The analysis findings will then be evaluated and presented in light of their implications for statistical and financial practices. The study will assess how using logarithmic equations can improve the annual average growth and period computation when other parameters are known.

RESULT AND DISCUSSION

Conventional Calculation of Interest Rate

The time value of money is predicated on the belief that one dollar now is worth something different than a dollar tomorrow (Chancellor, 2022; Howard & Schwartz, 2022; Papazian, 2022). The primary reason for this concept is that money received now can be saved or invested now and earn interest or a return, resulting in more money in the future. Therefore, receiving money in the present is preferred over receiving it in the future.

Investing funds or saving in interest-bearing accounts and investments allows their worth to appreciate over time as interest income accumulates or returns on investments are realized. This notion is termed future value (FV), which denotes the increased worth of a particular sum of money at a later date. The calculation of future value (FV) is contingent upon the type of interest applied to the investment, either simple or compound interest. The computation of future value typically uses compound interest, which incorporates interest on interest in its calculation. Compound interest takes the total amount due at the end of each year and adds it to the principal and interest to calculate interest for the following year. The subsequent equation determines the compound interest's future value from known future value (FV), present value (PV), interest rate (k), and periods (n) (Brealy et al., 2023; Trofimov, 2019; Joshi, 2019).

$$FV = PV + (1+k)^n \dots\dots\dots(\text{Equation 2})$$

Description:

- FV = Future value
- PV = Present value
- k = interest rate
- n = period

Example 1. A \$500 deposit is placed in a bank that pays 12.3 percent annual interest. What will the amount of the money, including interest, be worth after three years?

The following computation will be obtained using Equation 1, shown earlier.

$$FV = 500 + (1+12,3)^3$$

$$FV = 708,12$$

Therefore, when a \$500 deposit is placed in a bank that pays 12,3 percent annual interest, the nominal amount of the deposit, including annual compounding interest after three years, will be \$708,12. The issue arises when the aforementioned case is converted to the opposite.

Example 2. A deposit worth \$500 is deposited; after three years, it is worth 708,12. What is the average annual rate of return received?

Several scholarly works in finance and accounting (e.g., Biegel, 2022; Nagel, 2021; Keiso et al., 2016) have proposed applying compounding interest tables to address the aforementioned calculations using the following equation.

$$FVIF_{7\%,3} = PV/FV \dots\dots\dots(\text{Equation 3})$$

$$FVIF_{7\%,3} = 500/708,12$$

$$FVIF_{7\%,3} = 0,7061$$

Using the method described above, the present value interest factor table, which is shown in Appendix 1, will be searched for the value that is the most similar to the one being sought. In the third year's row, the value of 0,7061 will be determined to fall somewhere in the range of 0,712 at an interest rate of 12% and 0,693 at an interest rate of 13%. The interest rate that was found falls somewhere in the range of 12% to 13%.

Several considerations must be addressed when considering the aforementioned mathematical method. The aforementioned computation method yields findings of low precision, as it presents outcomes as a range. Consequently, the calculation results



presented as a range cannot be reinserted into the present or future value equation. Furthermore, calculations conducted with tables are limited to specific interest rates and timeframes. The table does not include any computations based on the interest rates of 12,5672% or 57,1246% as examples. Values displayed in tabular format are generally rounded to three or four decimal places. Consequently, computations necessitating data with more precision are challenging to accomplish.

In continuation to the above approach, several accounting and finance works have proposed a more advanced method for avoiding range-based calculation results through an interpolation linear calculation of the previously obtained range (e.g., Bigel, 2022; Cotter, 2021; Kieso et al., 2019). Returning to the original calculation, we found that at a 12% interest rate, the value of 0,7061 was between 0,712 and 0,693. The next step is to perform the linear interpolation computation.

$$\text{Interest Rate} = \text{Lower Rate} + \frac{\text{PFIV_Lower Rate} - \text{Calculated FVIF}}{\text{PVIF_Lower Rate} - \text{PFIV_Higher Rate}} \dots\dots(\text{Equation 4})$$

$$\text{Interest Rate} = 12\% + \frac{0,7120 - 0,7061}{0,7120 - 0,6930}$$

$$\text{Interest Rate} = 12,310526\%$$

From the analysis of the calculation approach above, it is worth noting that scenarios involving ranges with table calculations have been addressed by calculations utilizing linear interpolation. Nevertheless, further investigation reveals that the acquired results approximate the actual value and do not offer highly accurate outcomes. In the given scenario, a more accurate computation would yield a value of 12,3%; instead, the linear interpolation method yields a result of 12,310526. A discrepancy of 0,01526% is thus apparent.

The aforementioned difference arises from at least two factors. First, the interpolation calculation is linear (Jang et al., 2025; Wang et al., 2022; Lukas et al., 2021), whereas the calculation of the time value of the money model is nonlinear (Biegel, 2022; Oganezov, 2006). Linear interpolation may be beneficial for estimating interest in the time value of money calculations when a particular calculation cannot perform more precise measurements. However, inaccuracy arises from the nonlinear relationship between discount and compound interest rates and their respective multipliers. In the presence of an exponent, a linear relationship cannot be achieved; instead, a curvilinear outcome will result (Biegel, 2022).

The second reason for the disparity that has been seen is the utilization of table values and decimal rounding (Nagel, 2021; White et al., 2020; Brechner & Bergeman, 2020), depending on the table utilized. Notably, the value of 0,712, obtained by rounding for three decimals, is obtained at an interest rate of 12%. It was also observed that the same pattern occurred when the interest rate was 13%, and the value was 0,693. However, those values are rounded, which might lead to result deviation.

Proposed Algorithmic Interest Rate Calculation

The study proposes computing using mathematical operations and equations. This approach is an alternative to the linear interpolation process discussed previously.

Logarithmic Computation to Calculate Interest Rate

This study applies the following mathematical operations using logarithmic methods to calculate Example 2.

$$\begin{aligned} FV &= PV (1 + k)^n \\ 708,12 &= 500 (1 + k)^3 \\ (1 + k)^3 &= 1,4162 \end{aligned}$$

Subsequently, each side of the segment will undergo multiplication by the logarithm, resulting in the following computation.

$$\begin{aligned} \text{Log } (1 + k)^3 &= \text{log } 1,4162 \\ 3 \text{ Log } (1 + k)^3 &= 0,151139 \\ \text{Log } (1 + k)^3 &= 0,151139/3 \\ \text{Log } (1 + k) &= 0,05038 \end{aligned}$$

The inverse logarithm of each segment is subsequently multiplied by each segment, resulting in the following calculations.

$$\begin{aligned} 10^{\text{Log } (1 + k)} &= 10^{0,05038} \\ (1 + k) &= 1,123 \\ k &= 12,3\% \end{aligned}$$

To verify the accuracy of the above calculation, the results can be validated into the fundamental equation of future value or present value. The results of the subsequent calculations are verified with a high degree of precision.

$$\begin{aligned} FV &= PV (1 + k)^n \\ 708,12 &= 500 (1 + 12,3)^3 \\ 708,12 &= 708,12 \end{aligned}$$

Developing Algorithmic Equation to Calculate Interest Rate

The study formulates the mathematical procedures described above into an equation using the following computation.

$$\begin{aligned} FV &= PV (1 + k)^n \\ (1 + k)^n &= FV/PV \\ \text{Log } (1 + k)^n &= \text{Log}(FV/PV) \\ n \text{ Log}(1 + k) &= \text{log}(FV/PV) \\ \text{Log}(1 + k) &= \text{log}[FV/PV]/n \\ 10^{\text{Log}(1 + k)} &= 10^{\text{log}[FV/PV]/n} \\ (1+k) &= 10^{\text{log}[FV/PV]/n} \\ k &= 10^{\text{log}[FV/PV]/n} - 1 \end{aligned}$$

Therefore, the interest rate for a single payment of compounding interest within the time value of money framework, when the future value (FV), present value (PV), and time (n) are provided, can be determined using the subsequent equation.

$$k = 10^{\text{log}[FV/PV]/n} - 1 \dots\dots\dots \text{(Equation 5)}$$

Description:

- k = interest rate
- FV = future value
- PV = present value
- n = period of time

The accuracy of the calculation will be demonstrated by computing Example 2 using Equation 5. The subsequent calculation yields a value of 12,3%, precisely similar



to the expected result. Therefore, it is logical to infer that the equation has been demonstrated to be effective.

$$k = 10^{\log[FV/PV]/n} - 1$$

$$k = 10^{\log[708,12/500]/3} - 1$$

$$k = 10^{\log[1,41624]/3} - 1$$

$$k = 10^{0,151137/3} - 1$$

$$k = 10^{0,050379} - 1$$

$$k = 1,123 - 1$$

$$k = 12,3\%$$

Conventional Calculation of Period

Similar to interest rate calculations, several scholarly works in finance and accounting (e.g., Biegel, 2022; Cotter, 2021; Keiso et al., 2016) have suggested using compounding interest tables to find out the period when the present value (PV), future value (FV), and interest rate are known (k).

The study continues to calculate the period from a single payment of compounding interest time value of money when other parameters are known and modify Example 2 to the subsequent example.

Example 3. A deposit of \$500 is made with an annual interest rate of 10%. What is the period required for the deposit to accumulate to \$1,000?

The conventional approach suggests solving the case in Example 3 using financial tables and the following calculations.

$$FVIF_{10\%,?yr} = PV/FV$$

$$FVIF_{10\%,?yr} = 500/1.000$$

$$FVIF_{10\%,?yr} = 0.50$$

This method, included in Appendix 1, will discover the value that most closely corresponds with the present value interest factor table. The two numbers discovered in the column for the 10% interest rate are 0.513 and 0.467, corresponding to the period between 7 and 8 years.

Several issues must be considered in conjunction with the calculation of interest rates. First, the above calculation procedure does not yield high-precision calculation results, as it only provides a range. Consequently, the present value or future value equation cannot be used to verify the calculation results only in the form of a range. Furthermore, the table is restricted to specific periods; for instance, it does not provide calculations for 7.12345 years. The results presented in table form are typically adjusted to three or four decimal places. Therefore, obtaining the results of calculations that necessitate a higher degree of precision is challenging.

Financial and accounting literature (e.g., Bigel, 2022; Cotter, 2021; Kieso et al., 2019) provides a more advanced approach that involves an interpolation linear calculation of the previously obtained range. This computation is done to avoid the calculation results being presented as ranges. Referring back to the computation presented before, it was discovered that the value of 0.5 ranged from 0.513 over seven years to 0.467 over eight years. After that, the linear interpolation calculation described below is carried out.

$$\text{Period} = \text{Lower Period} + \frac{\text{PFIV_Lower Period} - \text{Calculated Period}}{\text{PFIV_Lower Period} - \text{PFIV_Higher Period}} \dots\dots(\text{Equation 6})$$

<http://doi.org/10.25273/jap.v13i3.21770>

$$\text{Period} = 7 + \frac{0,513 - 0,5}{0,513 - 0,467}$$

$$\text{Period} = 7,282609$$

Upon examining the aforementioned calculation procedure, it is evident that linear interpolation has effectively addressed instances involving ranges inside tabular computations. Nevertheless, a more thorough examination reveals that the results are approximate and require more precision. The resulting discrepancy can be demonstrated using the derived number to the fundamental equation for future value.

$$\begin{aligned} \text{FV} &= \text{PV} (1 + k)^n \\ 1,000 &= 500 (1 + 10\%)^{7,282609} \\ 1,000 &= 500 \times 2,00192 \\ 1,000 &= 1.000,96 \end{aligned}$$

The aforementioned results are indeed proximate to the anticipated value. Nonetheless, there exists a discrepancy of 0,96. Similar to the rationale provided in the interest rate computation, the aforementioned discrepancy arises from at least two factors. First, the interpolation computation is linear, whereas the time value of money calculation is nonlinear. The resulting difference could be more evident since the desired interest rate and duration fall within a narrow range of decimal units. The second reason for the mismatch is using table values rounded to three or four digits, contingent upon the specific table employed—the figure of 0,513 in period 7 results from rounding to three decimal places. A similar pattern was observed at a value of 0,467 over an 8-year duration. This rounding may result in differences in the calculation outcomes.

Logarithmic Computation to Calculate Period

The study proposes the application of the following mathematical operations using logarithmic methods to calculate Example 3.

$$\begin{aligned} \text{FV} &= \text{PV} (1 + k)^n \\ 1.000 &= 500 (1 + 0,10)^n \\ (1 + 0,10)^n &= 1.000/500 \\ (1,10)^n &= 2 \end{aligned}$$

Then, each segment side will be multiplied by the logarithm, and the following calculation is obtained.

$$\begin{aligned} \text{Log} (1,10)^n &= \text{log}(2) \\ n \text{ Log}(1,1) &= \text{log}(2) \\ 0,0141393n &= 0,30103 \\ n &= 0,30103/0,0141393 \\ n &= 7,27 \text{ year} \end{aligned}$$

Therefore, if the annual interest rate is 10%, a \$500 deposit will become \$1.000 in 7,27 years.

To verify the accuracy of the above calculation, the results can be re-examined using the fundamental equation of future value or present value. The results of the subsequent calculations are verified with a high degree of precision.

$$\begin{aligned} \text{FV} &= \text{PV} (1 + k)^n \\ 1.000 &= 500 (1 + 10\%)^{7,27} \end{aligned}$$





$$1.000 = 500 \times 2$$

$$1.000 = 1.000$$

Developing Algorithmic Equation to Calculate Period

The study proceeds to formulate the above mathematical procedures into an equation using the following computation.

$$FV = PV (1 + k)^n$$

$$(1 + k)^n = FV/PV$$

$$n \text{ Log}(1 + k) = \text{log}(FV/PV)$$

$$n = \frac{\text{log}(FV/PV)}{\text{log}(1+k)} \dots\dots\dots(\text{Equation 7})$$

- Description:
- n = periods
 - FV = future value
 - PV = present value
 - k = interest rate

To verify the accuracy of the earlier computation, Equation 7 is used to calculate Example 3 mentioned above. The following computation yields a value of 7.27 years, matching the predicted result, and the formula can be considered valid.

$$n = \frac{\text{log}(FV/PV)}{\text{log}(1 + k)}$$

$$n = \frac{\text{log}(1.000/500)}{\text{log}(1 + 10\%)}$$

$$n = \frac{\text{log}(2)}{\text{log}(1,1)}$$

$$n = \frac{0,30103}{0,41393}$$

$$n = 7,27$$

Advantages of Using Algorithms in Calculating Interest Rate and Period

The results of the above computation demonstrate that, given other parameters, algorithmic calculation is another approach for calculating interest rates and periods for a single payment's time value of money. Nevertheless, upon closer examination, there are various benefits to utilizing algorithms for mathematical operations and calculations rather than the commonly used table tools in financial management literature and the linear interpolation approach. These include [1] the fact that users can insert the known numbers into the available equations to obtain the desired results; [2] the fact that equations can be utilized with a calculator or computer, eliminating the need for a financial table; and [3] the fact that the calculated results are not presented as a range. [4] Equations are more reliable than other issue-solving methods since they yield more precise conclusions and are not restricted to a specific range of interest rates or periods.

Application of the Equations to Statistics

When the present and future values are known, the earlier equations can be utilized in various financial and accounting applications associated with the interest rate and the period. However, in addition to their use in the business context, these two equations can also be utilized in statistics fields to find the annual average of

growth rates by employing the compounding growth pattern. When this occurs, the growth from the previous year is included to compute the growth for the following year.

Determining the Annual Compounded Average Growth Rate

Example 4. In the year 2000, the company had sales of \$1.000.000. After five years, its sales increased to \$2.000.000. Therefore, the company has grown by 100% over five years.

This conclusion can be drawn through the following computations.

$$\text{Growth} = \frac{\text{FV} - \text{PV}}{\text{PV}} \dots\dots\dots(\text{Equation 8})$$

$$\text{Growth} = \frac{2.000.000 - 1.000.000}{1.000.000}$$

$$\text{Growth} = 100\%$$

However, the aforementioned technique cannot be applied to determining the average yearly growth. In other words, despite a total growth of 100% over five years, the average annual growth is less than 20%, derived by dividing the total growth by the duration of the growth period. Table 1 presents the annual computation utilizing the growth average, culminating in a final sales value of \$2.488.320 in the fifth year instead of the result of \$2.000.000. Therefore, the annual average growth rate cannot be determined by dividing the average growth rate by the number of years of growth.

Table 1. Computation of Growth Using Simple Calculation

| Year | Sales before Growth | Growth | Sales after Growth |
|------|---------------------|--------|--------------------|
| 2000 | 1.000.000 | 20% | 1.200.000 |
| 2001 | 1.200.000 | 20% | 1.440.000 |
| 2002 | 1.440.000 | 20% | 1.728.000 |
| 2003 | 1.728.000 | 20% | 2.073.600 |
| 2004 | 2.073.600 | 20% | 2.488.320 |

In light of the discrepancies observed in the preceding calculation, the study suggests the usage of Equation 5.

$$k = 10^{\log[\text{FV}/\text{PV}]/n} - 1 \dots\dots\dots (\text{Equation 5})$$

Description:

- k = average annual growth
- FV = value after several years
- PV = initial value to growth
- n = period

The results of the calculations are as follows:

$$k = 10^{\log[2.000.000/1.000.000]/5} - 1$$

$$k = 10^{\log[2]/5} - 1$$

$$k = 10^{0,30103/5} - 1$$

$$k = 10^{0,06021} - 1$$

$$k = 1,148698 - 1$$

$$k = 0,148698$$

The calculated average yearly growth is 14,8698% based on the aforementioned equation. Table 2 demonstrates the verification of the computation findings through





the fifth year. The computation reveals a discrepancy of \$3 from \$2,000,000, attributable to rounding the growth rate to four decimal places. The result in Table 2 contrasts significantly with the \$488.320 discrepancy noted in the prior calculation in Table 1.

Table 2. Computation of Growth Using Annual Compounded Average Growth Rate

| Year | Sales before Growth | Growth | Sales after Growth |
|------|---------------------|----------|--------------------|
| 2000 | 1.000.000 | 14,8698% | 1.148.698 |
| 2001 | 1.148.698 | 14,8698% | 1.319.507 |
| 2002 | 1.319.507 | 14,8698% | 1.515.715 |
| 2003 | 1.515.715 | 14,8698% | 1.741.099 |
| 2004 | 1.741.099 | 14,8698% | 1.999.997 |

Calculating Period in Statistical Growth

This study also suggests the application of Equation 7 to determine the period when other parameters are available.

$$n = \frac{\log(FV/PV)}{\log(1+k)} \dots\dots\dots(\text{Equation 7})$$

Description:

- n = periods
- FV = value after several years
- PV = initial value to growth
- k = average annual growth

This study modifies the scenario in Example 4 above for illustrative purposes in the calculation.

Example 5. A company generated revenues of \$1,000,000 in the year 2000. Given an average yearly growth rate of 14.8698%, what is the required period for the company's revenues to reach \$2,000,000?

With the following computations, the previously described algorithm approach in Equation 7 may solve the aforementioned case. The 5 years result derived from the equation is accurate. Therefore, the equation is validated for use.

$$n = \frac{\log(2.000.000/1.000.000)}{\log(1 + 14,8698\%)}$$

$$n = \frac{\log(2)}{\log(1,148698)}$$

$$n = \frac{0,30103}{0,060206}$$

$$n = 5$$

CONCLUSION

According to the study's findings, algorithmic computation efficiently determines the interest rate and the period when both the present and future values exist. The two equations formulated using an algorithmic method offer alternatives to the traditional dependence on financial tables and linear interpolation commonly used in the existing literature. The equations are validated and substantiated upon reevaluation of the fundamental calculation of the time value of money, which results in results that are far more precise than those obtained using the conventional method. The measurement can also be utilized in statistics to compute the compounded average annual growth or growth period when other parameters are available.

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Appendix 1. Present value interest factor for \$1 discounted at k percent for n periods:
 $PVIF_{k,n} = (1/(1+k)^n)$

| Period | 1% | 2% | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% | 11% | 12% | 13% | 14% | 15% | 16% | 20% | 24% | 25% | 30% |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 | 0.9901 | 0.9804 | 0.9709 | 0.9615 | 0.9524 | 0.9434 | 0.9346 | 0.9259 | 0.9174 | 0.9091 | 0.9009 | 0.8929 | 0.8850 | 0.8772 | 0.8696 | 0.8621 | 0.8333 | 0.8065 | 0.8000 | 0.7692 |
| 2 | 0.9803 | 0.9612 | 0.9426 | 0.9246 | 0.9070 | 0.8900 | 0.8734 | 0.8573 | 0.8417 | 0.8264 | 0.8116 | 0.7972 | 0.7831 | 0.7695 | 0.7561 | 0.7432 | 0.6944 | 0.6504 | 0.6400 | 0.5917 |
| 3 | 0.9706 | 0.9423 | 0.9151 | 0.8890 | 0.8638 | 0.8396 | 0.8163 | 0.7938 | 0.7722 | 0.7513 | 0.7312 | 0.7118 | 0.6931 | 0.6750 | 0.6575 | 0.6407 | 0.5787 | 0.5245 | 0.5120 | 0.4552 |
| 4 | 0.9610 | 0.9238 | 0.8885 | 0.8548 | 0.8227 | 0.7921 | 0.7629 | 0.7350 | 0.7084 | 0.6830 | 0.6587 | 0.6355 | 0.6133 | 0.5921 | 0.5718 | 0.5523 | 0.4823 | 0.4230 | 0.4096 | 0.3501 |
| 5 | 0.9515 | 0.9057 | 0.8626 | 0.8219 | 0.7835 | 0.7473 | 0.7130 | 0.6806 | 0.6499 | 0.6209 | 0.5935 | 0.5674 | 0.5428 | 0.5194 | 0.4972 | 0.4761 | 0.4019 | 0.3411 | 0.3277 | 0.2693 |
| 6 | 0.9420 | 0.8880 | 0.8375 | 0.7903 | 0.7462 | 0.7050 | 0.6663 | 0.6302 | 0.5963 | 0.5645 | 0.5346 | 0.5066 | 0.4803 | 0.4556 | 0.4323 | 0.4104 | 0.3349 | 0.2751 | 0.2621 | 0.2072 |
| 7 | 0.9327 | 0.8706 | 0.8131 | 0.7599 | 0.7107 | 0.6651 | 0.6227 | 0.5835 | 0.5470 | 0.5132 | 0.4817 | 0.4523 | 0.4251 | 0.3996 | 0.3759 | 0.3538 | 0.2791 | 0.2218 | 0.2097 | 0.1584 |
| 8 | 0.9235 | 0.8535 | 0.7894 | 0.7307 | 0.6768 | 0.6274 | 0.5820 | 0.5403 | 0.5019 | 0.4665 | 0.4339 | 0.4039 | 0.3762 | 0.3506 | 0.3269 | 0.3060 | 0.2326 | 0.1789 | 0.1678 | 0.1226 |
| 9 | 0.9143 | 0.8368 | 0.7664 | 0.7026 | 0.6446 | 0.5919 | 0.5439 | 0.5002 | 0.4604 | 0.4241 | 0.3909 | 0.3606 | 0.3329 | 0.3075 | 0.2843 | 0.2630 | 0.1938 | 0.1443 | 0.1342 | 0.0943 |
| 10 | 0.9053 | 0.8203 | 0.7441 | 0.6756 | 0.6139 | 0.5584 | 0.5083 | 0.4632 | 0.4224 | 0.3855 | 0.3522 | 0.3220 | 0.2946 | 0.2697 | 0.2472 | 0.2267 | 0.1615 | 0.1164 | 0.1074 | 0.0725 |
| 11 | 0.8963 | 0.8043 | 0.7224 | 0.6496 | 0.5847 | 0.5268 | 0.4751 | 0.4289 | 0.3875 | 0.3505 | 0.3173 | 0.2875 | 0.2607 | 0.2366 | 0.2149 | 0.1954 | 0.1346 | 0.0938 | 0.0859 | 0.0558 |
| 12 | 0.8874 | 0.7885 | 0.7014 | 0.6246 | 0.5568 | 0.4970 | 0.4440 | 0.3971 | 0.3555 | 0.3186 | 0.2858 | 0.2567 | 0.2307 | 0.2076 | 0.1869 | 0.1685 | 0.1122 | 0.0757 | 0.0687 | 0.0429 |
| 13 | 0.8787 | 0.7730 | 0.6810 | 0.6006 | 0.5303 | 0.4688 | 0.4150 | 0.3677 | 0.3262 | 0.2897 | 0.2575 | 0.2292 | 0.2042 | 0.1821 | 0.1625 | 0.1452 | 0.0935 | 0.0610 | 0.0550 | 0.0330 |
| 14 | 0.8700 | 0.7579 | 0.6611 | 0.5775 | 0.5051 | 0.4423 | 0.3878 | 0.3405 | 0.2992 | 0.2633 | 0.2320 | 0.2046 | 0.1807 | 0.1597 | 0.1413 | 0.1252 | 0.0779 | 0.0492 | 0.0440 | 0.0254 |
| 15 | 0.8613 | 0.7430 | 0.6419 | 0.5563 | 0.4810 | 0.4173 | 0.3624 | 0.3152 | 0.2745 | 0.2394 | 0.2090 | 0.1827 | 0.1599 | 0.1401 | 0.1229 | 0.1079 | 0.0649 | 0.0397 | 0.0352 | 0.0195 |
| 16 | 0.8528 | 0.7284 | 0.6232 | 0.5339 | 0.4581 | 0.3936 | 0.3387 | 0.2919 | 0.2519 | 0.2176 | 0.1883 | 0.1631 | 0.1415 | 0.1229 | 0.1069 | 0.0930 | 0.0541 | 0.0320 | 0.0281 | 0.0150 |
| 17 | 0.8444 | 0.7142 | 0.6050 | 0.5134 | 0.4363 | 0.3714 | 0.3166 | 0.2703 | 0.2311 | 0.1978 | 0.1696 | 0.1456 | 0.1252 | 0.1078 | 0.0929 | 0.0802 | 0.0451 | 0.0258 | 0.0225 | 0.0116 |
| 18 | 0.8360 | 0.7002 | 0.5874 | 0.4936 | 0.4155 | 0.3503 | 0.2959 | 0.2502 | 0.2120 | 0.1799 | 0.1528 | 0.1300 | 0.1108 | 0.0946 | 0.0808 | 0.0691 | 0.0376 | 0.0208 | 0.0180 | 0.0089 |
| 19 | 0.8277 | 0.6864 | 0.5703 | 0.4746 | 0.3957 | 0.3305 | 0.2765 | 0.2317 | 0.1945 | 0.1635 | 0.1377 | 0.1161 | 0.0981 | 0.0829 | 0.0703 | 0.0596 | 0.0313 | 0.0168 | 0.0144 | 0.0068 |
| 20 | 0.8195 | 0.6730 | 0.5537 | 0.4564 | 0.3769 | 0.3118 | 0.2584 | 0.2145 | 0.1784 | 0.1486 | 0.1240 | 0.1037 | 0.0868 | 0.0728 | 0.0611 | 0.0514 | 0.0261 | 0.0135 | 0.0115 | 0.0053 |
| 21 | 0.8114 | 0.6598 | 0.5375 | 0.4388 | 0.3589 | 0.2942 | 0.2415 | 0.1987 | 0.1637 | 0.1351 | 0.1117 | 0.0926 | 0.0768 | 0.0638 | 0.0531 | 0.0443 | 0.0217 | 0.0109 | 0.0092 | 0.0040 |
| 22 | 0.8034 | 0.6468 | 0.5219 | 0.4220 | 0.3418 | 0.2775 | 0.2257 | 0.1839 | 0.1502 | 0.1228 | 0.1007 | 0.0826 | 0.0680 | 0.0560 | 0.0462 | 0.0382 | 0.0181 | 0.0088 | 0.0074 | 0.0031 |
| 23 | 0.7954 | 0.6342 | 0.5067 | 0.4057 | 0.3256 | 0.2618 | 0.2109 | 0.1703 | 0.1378 | 0.1117 | 0.0907 | 0.0738 | 0.0601 | 0.0491 | 0.0402 | 0.0329 | 0.0151 | 0.0071 | 0.0059 | 0.0024 |
| 24 | 0.7876 | 0.6217 | 0.4919 | 0.3901 | 0.3101 | 0.2470 | 0.1971 | 0.1577 | 0.1264 | 0.1015 | 0.0817 | 0.0659 | 0.0532 | 0.0431 | 0.0349 | 0.0284 | 0.0126 | 0.0057 | 0.0047 | 0.0018 |
| 25 | 0.7798 | 0.6095 | 0.4776 | 0.3751 | 0.2953 | 0.2330 | 0.1842 | 0.1460 | 0.1160 | 0.0923 | 0.0736 | 0.0588 | 0.0471 | 0.0378 | 0.0304 | 0.0245 | 0.0105 | 0.0046 | 0.0038 | 0.0014 |
| 30 | 0.7419 | 0.5521 | 0.4120 | 0.3083 | 0.2314 | 0.1741 | 0.1314 | 0.0994 | 0.0754 | 0.0573 | 0.0437 | 0.0334 | 0.0256 | 0.0196 | 0.0151 | 0.0116 | 0.0042 | 0.0016 | 0.0012 | * |
| 35 | 0.7059 | 0.5000 | 0.3554 | 0.2534 | 0.1813 | 0.1301 | 0.0937 | 0.0676 | 0.0490 | 0.0356 | 0.0259 | 0.0189 | 0.0139 | 0.0102 | 0.0075 | 0.0055 | 0.0017 | 0.0005 | * | * |
| 36 | 0.6989 | 0.4902 | 0.3450 | 0.2437 | 0.1727 | 0.1227 | 0.0875 | 0.0626 | 0.0449 | 0.0323 | 0.0234 | 0.0169 | 0.0123 | 0.0089 | 0.0065 | 0.0048 | 0.0014 | * | * | * |
| 40 | 0.6717 | 0.4529 | 0.3066 | 0.2083 | 0.1420 | 0.0972 | 0.0668 | 0.0460 | 0.0318 | 0.0221 | 0.0154 | 0.0107 | 0.0075 | 0.0053 | 0.0037 | 0.0026 | 0.0007 | * | * | * |
| 50 | 0.6080 | 0.3715 | 0.2281 | 0.1407 | 0.0872 | 0.0543 | 0.0339 | 0.0213 | 0.0134 | 0.0085 | 0.0054 | 0.0035 | 0.0022 | 0.0014 | 0.0009 | 0.0006 | * | * | * | * |

