

The importance of mathematical disposition and procedural-conceptual knowledge for elementary school teachers

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Abstract: The success of learning mathematics in elementary schools is strongly influenced by the level of mathematical disposition, procedural knowledge, and conceptual knowledge of a teacher. This study follows an explanatory sequential design with the objectives of (1) investigating the effect of mathematical disposition on elementary school teachers' mathematical knowledge; and (2) describing the relationship between mathematical disposition, procedural knowledge, and conceptual knowledge in the formation of teachers' mathematical knowledge in problem-solving learning in elementary schools. Data collection through tests, questionnaires, and interviews. The test and questionnaire data were analyzed through linear regression statistical tests. Interview data were analyzed descriptively. Test the validity of interview data through triangulation of sources and theories. The results showed an effect of the level of mathematical disposition on mathematical knowledge (procedural and conceptual knowledge). Teachers with a high level of disposition have high procedural and conceptual knowledge. Meanwhile, teachers with low disposition level have lower procedural and conceptual knowledge. A positive mathematical disposition is a relationship between mathematical disposition procedural knowledge, and conceptual knowledge. It has implications for the formation of mathematical knowledge (procedural and conceptual knowledge) of a teacher in problem-solving learning. Also, the mathematical disposition is a prerequisite that supports the formation of mathematical knowledge (procedural and conceptual knowledge).

Keywords: Conceptual knowledge, Mathematical disposition, Procedural knowledge

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INTRODUCTION

Elementary school teachers are the foundations (first) formally in acquiring students' knowledge, skills, and attitudes.. The professional task of teaching teachers at the elementary school level is not an easy task. Elementary school teachers must master several subjects, including mathematics. This condition requires elementary school teachers to equip themselves with good professional and pedagogical skills (Gronseth et al., 2010; Kim et al., 2019).

At the elementary school level, children (students) learn mathematics by imitating the teacher (Blazar & Kraft, 2017; Salisu & Ransom, 2014). Therefore, the teacher must provide mathematical concepts according to the correct theorems and problem-solving procedures (Ahmad Al-Khateeb, 2016; Mulyono et al., 2019). It is also important for teachers to use their knowledge with accurate mathematical language so that they are familiar with the language of mathematics and can understand it. The teacher's use of accurate mathematical language will prevent or reduce misconceptions in learning (Aliustaoğlu et al., 2018; Kusmaryono et al., 2020). Therefore, to teach problem-solving skills to students, of course, teachers are required to have mathematical knowledge. This mathematical knowledge includes conceptual and procedural knowledge (Otun & Olaoye, 2019).

The indicator of the success of learning mathematics in school is when students have a positive attitude towards mathematics (mathematical disposition), students have the ability to think logically, and students' creativity in developing mathematical procedures (Julia et al., 2020). Consequently, primary school teachers must have strong and deep conceptual and procedural knowledge to shape students' mathematical knowledge (Fitriati et al., 2016; Kadijevich, 2018).

Conceptual and procedural knowledge plays an important role in the ability of teachers to help develop children's mathematical understanding (Markovits & Patkin, 2020; Boonen et al., 2013; Boonen et al., 2016). To be competent in learning mathematics, it must involve conceptual and procedural knowledge and the relationship between these two types of mathematical knowledge (Rittle-Johnson et al., 2016).

Conceptual knowledge is a person's mental representation of the domain's principles (Rittle-Johnson et al., 2016). Conceptual knowledge represents skillful knowledge and 'drive' around a particular network, the elements of which can be concepts, and rules (algorithms, procedures, etc.). Conceptual knowledge is divided into five indicators, namely identifying related facts; recognize examples and non-examples; interpreting signs, symbols, and terms; manipulating related ideas; perfecting the relationship between concepts and principles (Otun & Olaoye, 2019).

Procedural knowledge is often reflected as a person's ability to relate an algorithmic process to a given problem situation, work on the algorithm correctly, and communicate the results of the algorithm in the context of the problem (Kadijevich, 2018). The opinion of other experts states that procedural knowledge is the ability to carry out a sequence of actions, rules, and procedures to solve problems (Rittle-Johnson et al., 2016). One of the characteristics of procedural knowledge is that there is a sequence of steps that will be taken, namely, after a step, the next step will be followed (Otun & Olaoye, 2019; Rittle-Johnson et al., 2016).

Procedural knowledge includes various number algorithms in mathematics which are used to find more precise results precisely (Kadijevich, 2018). Procedural knowledge leads to reading and creating graphing and tables, carrying out geometric constructions, and displaying non-computational skills such as rounding and classifying (Otun & Olaoye, 2019). Procedural knowledge tends to be more computational and knowledge about the steps to identify mathematical objects, algorithms, and definitions. These steps include identifying and solving problems (Kadijevich, 2018).

The difference between conceptual and procedural aspects of learning mathematics

has been discussed in the research literature. Researchers note that (1) procedural knowledge usually requires automatic and unconscious steps, whereas conceptual knowledge usually requires conscious thinking; (2) procedural knowledge may also involve some conscious thinking (for example when a person skillfully combines two rules without knowing why they work (Kadijevich, 2018; Österman & Bråting, 2019; Otun & Olaoye, 2019).

Teaching mathematics for procedural knowledge can mean presenting previously formulated definitions, notations, and procedures without providing a meaningful context for the concepts and methods involved (Otun & Olaoye, 2019). Teaching for conceptual understanding begins by presenting reasoning problems that involve the meaning of mathematical objects (which students have) so that there is a connection to their previous knowledge (Kadijevich, 2018; Rittle-Johnson et al., 2016). However, an expert argues that it is necessary to study procedures first to a conceptual understanding concerning making meaning in mathematics (Otun & Olaoye, 2019).

Dispositions are beliefs and tendencies that encourage someone to respond or act (Feldhaus, 2014). What someone thinks will be actualized as an action. Actions will be directed if a person's mental function is in good condition and under control (Beyers, 2011; Feldhaus, 2014). Psychologically, a disposition is related to one's mental function (Anku, 1996; Feldhaus, 2014). Disposition is not a behavior but rather a determinant of behavior and represents the ways how a person perceives things. Disposition does not only refer to attitudes but tends to think and act positively (Feldhaus, 2014).

Some literature defines mathematical dispositions as one's beliefs and tendencies to behave about mathematics as reasonable, useful, and valuable (Beyers, 2011; Feldhaus, 2014; Kusmaryono, et al., 2019). Another expert added that mathematical disposition includes a willingness to take risks and explore solutions to various problems, persistence to solve challenging problems, responsible for reflecting on work results, appreciating the strength of mathematics, willingness to ask questions and propose other mathematical ideas, willing to try various ways to explore mathematical concepts, have confidence in their abilities, and see problems as challenges (Anku, 1996; Mueller & Hindin, 2011).

Mathematics learning which is supported by the productive mathematical disposition of the teacher will increase creativity and learning innovation. This positive mathematical disposition from the teacher will give positive energy to students' learning activities and mathematical dispositions. Students with positive mathematical dispositions generally will increase self-confidence and be motivated to learn mathematics (Beyers, 2011; Kusmaryono, et al., 2019).

This research was conducted with the aim of (1) investigating the effect of mathematical disposition on elementary school teachers' mathematical knowledge; and (2) describing the relationship between mathematical disposition, procedural knowledge, and conceptual knowledge in the formation of teachers' mathematical knowledge in problem-solving learning in elementary schools.

Given the importance of mathematical disposition and mathematical knowledge (conceptual and procedural knowledge) in teacher performance in primary schools, this certainly deserves to be studied in depth. The results of this study are expected to contribute to teachers' understanding of the importance of mathematical disposition and mastery of conceptual and procedural knowledge in improving the quality of mathematics learning in elementary schools.

METHODS

Research Design

This study was designed using a combination of combining quantitative and qualitative methods with a sequential explanatory strategy. The sequential explanatory strategy is a

strategy that is applied by collecting and analyzing quantitative data in the first stage followed by the collection and analysis of qualitative data in the second stage, which is built on the results of the initial quantitative data (Castrol et al., 2010; Creswell, 2014).

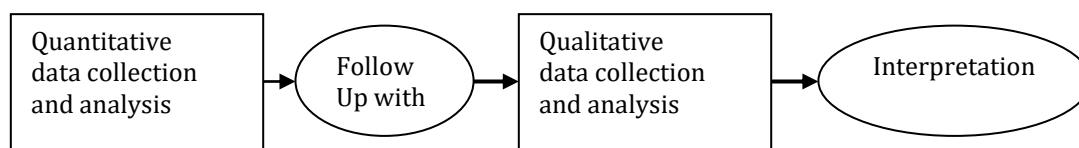


FIGURE 1. Explanatory sequential design

Instruments

Research instruments for obtaining data include test and non-test instruments (questionnaires and interviews). The test questions consist of 8 problem-solving items in the form of descriptions. The material for the test is elementary school mathematics which includes 4 items involving conceptual knowledge (CK) and 4 items involving procedural knowledge (PK).

TABLE 1. Indicator of problem-solving test

Question Number	Indicator Problem Solving test	Type of knowledge	Category
1	Determining distance, time, and speed	PK	difficult
2	Calculating the area of a circle	CK	quite difficult
3	Calculating the volume of water in the tube	PK	difficult
4	Interpreting statistical data	CK	quite difficult
5	Solving integer problems	PK	difficult
6	Solving mixed operations on rational numbers	CK	quite difficult
7	Solving social arithmetic problems	PK	difficult
8	determining the map image comparison scale	CK	quite difficult

The questionnaire is arranged according to the mathematical disposition (MD) indicator which consists of 20 statement items. All test instruments have been validated and recommended by a team of experts to be used as valid data collection tool. The instrument and questionnaire reliability test was carried out using the Cronbach's alpha test. Each reliability tests obtained Cronbach's alpha values of 0.714 and 0.770 with high-reliability criteria (Taber, 2018). The r table value was used for $n = 20$ with a significance of 0.444 (Isaac & Chikweru, 2018). With these results, the results of $r_{count} > r_{table}$ so that the test instruments and questionnaires developed and tested can be declared reliable. Interviews were conducted in a semi-structured manner with several main questions that had been prepared.

Participants

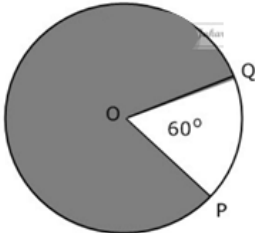
The population of this study was elementary school teachers in the city of Semarang, Indonesia. A total of 30 teachers from ten elementary schools were involved in this study. In general, teachers have 4 to 20 years of experience teaching mathematics in elementary schools and they have taught mathematics at different grade levels in elementary schools. Interview subjects were selected through purposive sampling and snowball techniques (Castrol, et al., 2010). Prepared interview questions can be developed in the field according to the conditions and needs of the study. Interviews aim to confirm and obtain in-depth data about the teacher's conceptual and procedural knowledge in solving

mathematical problems, as well as ensure the influence of mathematical dispositions on mathematical knowledge (conceptual and procedural) teachers in problem-solving.

Problem-solving test

The following shows two examples of the eight-item problem-solving tests that the teacher has to solve. Each item in this test represents a mathematical problem to measure aspects of conceptual and procedural knowledge.

TABLE 2. *Sample of problem-solving test*

Item	No.	Problems
Procedural problem	1.	Two cars A and car B move together from different cities 210 km away. Car A has a speed of 80 km/hour and car B is 60 km/hour. After how many minutes will car A and car B meet?
Conceptual problem	2.	<p>Look at the circle image below.</p>  <p>If the line length is $OQ = OP = 30$ cm, then calculate the area of the shaded circle!</p>

Data analysis

The quantitative data on the results of the mathematics knowledge test were analyzed using descriptive statistics. The data from the mathematical disposition questionnaire were converted into a Likert scale with a score range of 1 to 5. The level of mathematical disposition is measured based on criteria that have been developed in previous studies (Kusmaryono et al., 2019). Then a statistical test was carried out with simple linear regression analysis to determine the effect of mathematical disposition on mathematical knowledge (conceptual and procedural knowledge). Data from the interview collected by data reduction and coding then analyzed descriptively. Test the validity of qualitative data through source triangulation and theory triangulation (Castrol et al., 2010; Miles & Huberman, 2016). All data that has been collected investigated the relationship between data to find out whether the two data (quantitative and qualitative) contradictor reinforce each other.

RESULTS

MK and MD of elementary school teachers

The MK data is a combination of the average PK and CK scores of each test taker. The research data collected were tabulated in descriptive statistical tables which included PK, CK, and MD. The grouping of the MK test result data is divided into three based on the distribution of the data with a normal distribution, namely the upper group, middle group, and lower group. The upper group of the distribution of values in the curve region ($\bar{x} + 1\sigma < \text{score}$), the middle group of the distribution of values in the curve region ($\bar{x} - 1\sigma \leq \text{score} \leq \bar{x} + 1\sigma$), and the lower group of the distribution of values in the curve region ($\text{score} < \bar{x} - 1\sigma$). The statistical descriptions of the test result data are presented in

TABLE 3.**TABLE 3.** *Statistical of mathematical knowledge*

Groups	Number of Respondents	Mean Score			Criteria
		PK	CK	MK	
Top	11	88.33	83.33	85.83	high
Middle	13	73.18	73.64	73.41	moderate
Bottom	6	53.00	56.00	54.50	very low
Total	30	71.33	71.67	71.73	moderate

TABLE 4. *Statistical of mathematical dispositions*

Groups	Number of Respondents	Percentage (%)	Mean Score	Criteria
Top	11	37%	82.67	high
Middle	13	43%	73.94	moderate
Bottom	6	20%	61.60	low
Total	30	100%	72.77	moderate

TABLE 5. *The linear regression statistical test*

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Constant	11.692	10.912		1.07	.029
	Dispos_X	.825	.149	.723	5.53	.000

a. Dependent Variable: MathKnowldge_Y

The range of scores used as the assessment criteria are: a score of 90.0 - 100.0 (very high); score 80.0 - 89.9 (high); score 65.0 - 79.9 (medium); score 55.0 - 64.9 (low); and the mean value < 55.0 (very low). From Table 4 it can be seen that teachers in the upper group have an MK score of 85.83 (high), teachers in the middle group have an MK score of 73.41 (medium), and teachers in the lower group have an MK score of 54.50 (very low).

The data from the MD questionnaire that has been filled in by the respondent (teacher) is then processed and analyzed. The MD questionnaire consists of 20 statement items with a score range of 1 to 100, so the maximum score the respondents can obtain is 100. The statistical descriptions of the data from the results of the MD questionnaire are presented in **TABLE 4**.

The statistical distribution in **TABLE 4** shows that only 37% (11) of teachers have a high category of MD. Meanwhile, 63% (19) of teachers have a lower middle MD. Noting the statistical descriptions of the questionnaire results in **TABLE 4**, the researcher (R) needs to conduct interviews to obtain additional information about the respondents' MD. The following is a snippet of the researcher's interview with the representatives of each group of respondents (R = researchers; TT = teachers of the top group, TM = teachers of the middle group, and TB = teachers of the bottom group).

1st Interview

- R : Do you like math-related things?
 TT-22 : Yes, I am an elementary school teacher who has to learn a lot about math.
 TM-11 : Of course, because everything has to do with mathematics.
 TB-25 : Actually, math is not my area of expertise. But I was forced to teach it

- at school.
- R : Has studying math influenced your career so far?
- TT-22 : Yes, math demands that I can solve any problem.
- TM-11 : Yes, I feel like the teacher that students expect.
- TB-25 : Yes, I have to try hard to understand math well for my students in school.
- R : What do you do if you have difficulty learning mathematics (problem-solving)?
- TT-22 : I will try hard until I find a solution
- TM-11 : I feel like this is a challenge that I have to face
- TB-25 : Maybe I will ask a senior teacher.

The linear regression statistical test

To determine the effect of mathematical disposition (X) on teachers' mathematical knowledge (Y), a linear regression statistical test was performed. The results of the linear regression statistical test are shown in **TABLE 5**. In **TABLE 5** the output results show the significance (Sig.) = 0.000 less than 0.05, the null hypothesis is rejected. So it can be concluded that there is an effect of mathematical disposition (X) on mathematics knowledge (Y) of elementary school teachers.

Description of PK from Elementary School Teacher

Subject responses to PK aspects of mathematics problems have been carefully researched and analyzed. The quality of the responses was divided into three groups, namely the upper group, middle group, and lower group. Each group takes one example of the correct answer according to the aspect of knowledge being assessed. The following is an example of the results of the subject's answers (top group) in solving problems in the PK aspect of mathematics.

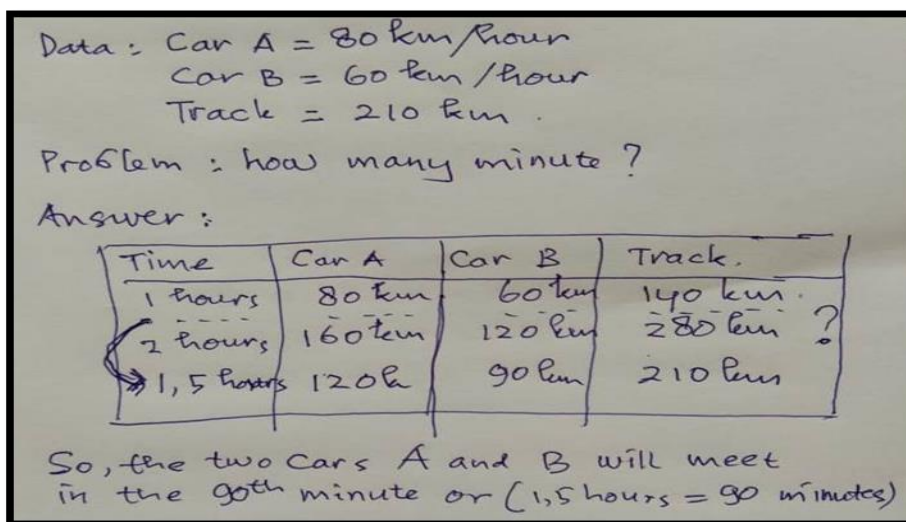


FIGURE 2. Answers to PK problems (TT-22)

After analyzing the answers of the subjects (TM-16) in **FIGURE 2**, it seems that the use of tables is quite helpful for the subjects in finding solutions, the researcher (R) conducted interviews with subjects in the upper group (TT-22). The purpose of the interview is to explore deeper information about PK which is applied in solving problems.

2nd Interview

- R : Why do you present the problem-solving procedure in a table?
- TT-22 : I believe that through this table students will easily understand.
- R : Why didn't you choose an answer within 1.5 hours?
- TT-22 : In the first hour, car A covers 80 km and car B 60 km. if the mileage is added to be 140 km. This means that there is still a path to go and the two cars have not yet met.
- R : What about the travel time 2 hours later?
- TT-22 : When 2 hours passed, cars A and B traveled 160 km + 120 km = 280 km. It means they have met before.
- R : Are you sure that solving through tables is easier for students to understand?
- TT-22 : Sure. Through this table, students can think realistically, because students can determine which procedure is right. In 1.5 hours, car A travels 120 km and car B 90 km. if you add up the total distance traveled 210 km. this means that the two cars meet during the 1.5-hour journey.

The following is an example of the results of the subject's answers (middle group) in solving mathematical problems on the PK aspects.

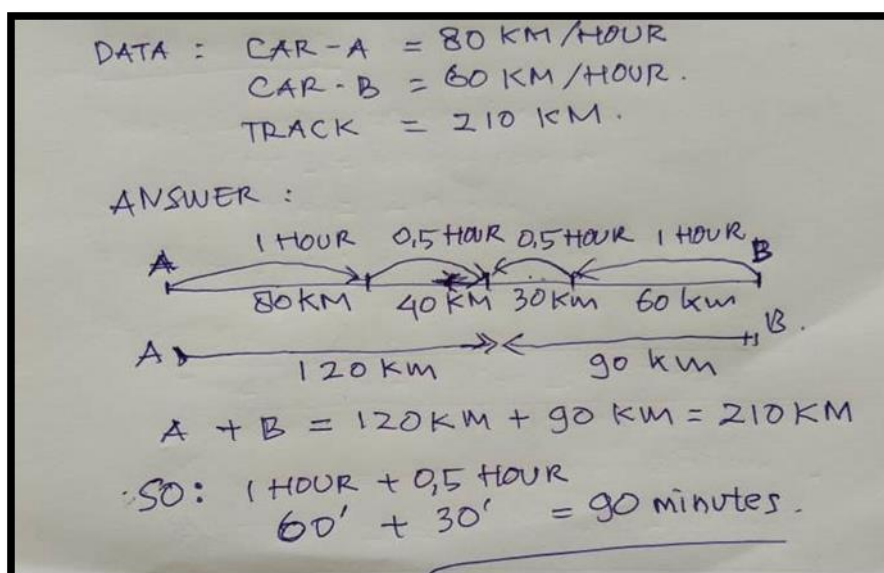


FIGURE 3. Answers to PK problems (TM-16)

Noting the response of the object (**FIGURE 3**) it seems that using line sketches is very effective, the researcher (R) conducted interviews with subjects in the middle group (TM-16) to dig deeper information about PK that was applied in solving problems.

3rd Interview

- R : Why can you solve this problem properly?
- TM-16 : I understand the issues being asked and the adequacy of the data
- R : How do you perform this troubleshooting procedure?
- TM-16 : Cars A and B move together in a straight line from opposite directions with different speeds.
- R : What rules did you use to solve the problem?
- TM-16 : Rule of operation for subtraction on rational numbers

- R : Where did you get the idea of the problem-solving procedure?
 TM-16 : Ideas from the travel experience of driving a car on the motorway
 R : Have your ideas been applied to classroom learning?
 TM-16 : I have taught the same way to my students in math class.

The following is an example of the results of the subject's answers (bottom group) in solving mathematical problems on the aspects of PK.

Handwritten mathematical solution for PK problems (TB-25):

Answer No. 1
 or $\Rightarrow = 210 \text{ km} : (80 + 60)$
 $\Rightarrow = 210 : 140$
 $\Rightarrow = 1,5 \text{ HOUR}$
 $\Rightarrow = 90 \text{ minute}$

FIGURE 4. Answers to PK problems (TB-25)

Paying attention to the response of the subject's answer which is quite surprising (correct answer) (**FIGURE 4**), the researcher (R) conducted interviews with subjects in the lower group (TB-25) to dig deeper information about PK that was applied in solving problems.

4th Interview

- R : Why don't you step through the troubleshooting procedure properly?
 TB-25 : That's all in my head. Distance divided by the speed
 R : Where did you get the idea of the problem-solving procedure?
 TB-25 : I just think of two kids eating long instant noodles from two directions at different speeds.
 R : Is every problem, you always think the same thing?
 TB-25 : I don't know, maybe it's just a coincidence that the idea came up.

Description of CK from Elementary School Teacher

The response of the subject's answers to mathematical problems in the aspect of CK has been carefully researched and analyzed. Paying attention to the answers of the subject (TT-22) (top group) in solving math problems in the aspect of CK.

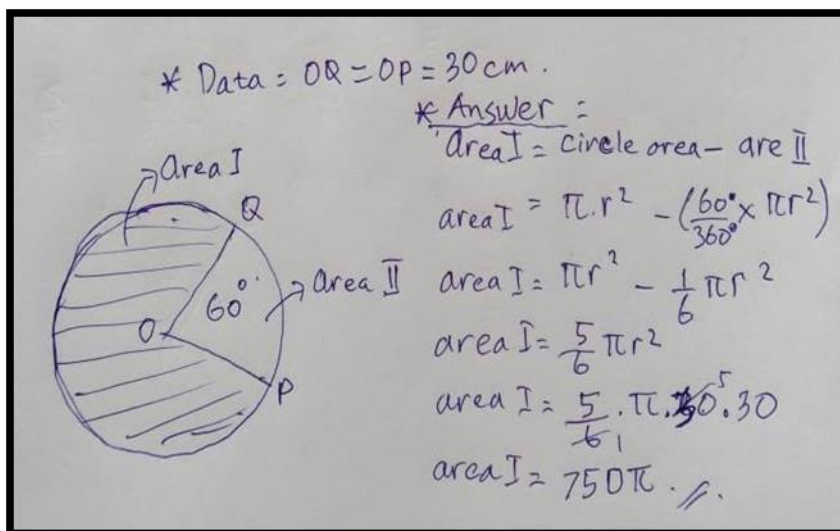


FIGURE 5. Answers to CK problems (TT-22)

Following up on the subject's response with the correct answer and using a well-structured completion step (FIGURE 5), the researcher (R) conducted interviews with the upper group subjects (TT-22) to dig deeper information about CK which was applied in solving math problems.

5th Interview

- R : Why don't you just count area I without counting area II?
 TT-22 : Conceptually area I is a complete circle and has been reduced by 60%.
 R : Are the same concepts taught to your students?
 TT-22 : I convey this concept to students so that students can relate their knowledge of the area of a circle that has been previously understood.

Paying attention to the answers of the subject (TM-11) (middle group) in solving math problems in the aspect of CK.

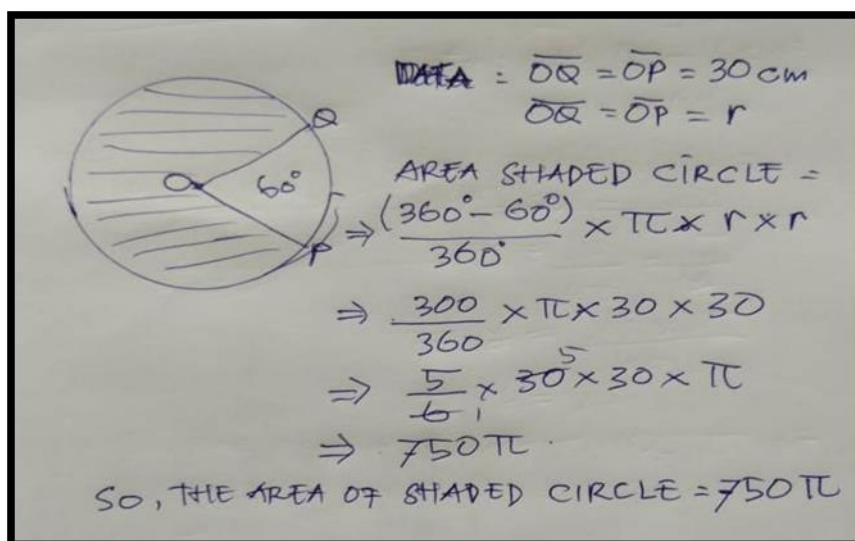


FIGURE 6. Answers to CK problems (TM-11)

Following up on the subject's response with the correct answer and a different way of solving it from other subjects (**FIGURE 6**), researcher (R) conducted interviews with middle group subjects (TM-11) to dig deeper into information about CK which was applied in solving math problems.

6th Interview

- R : What do you understand about the concept of shaded areas?
 TM-11 : The area of the circle minus the circumference of the circle
 R : Why don't you calculate the area of the circle minus the area of the radius?
 TM-11 : Focus on the shaded area is a more efficient step.
 R : What conceptual knowledge did you use to solve this problem?
 TM-11 : The concept of the area of a circle, the concept of percent, and the operations of division and multiplication of rational numbers
 R : Does your procedural knowledge contribute to solving these problems?
 TM-11 : What I thought first was how to solve this problem, then I used the concepts that I had mastered.

Paying attention to the answers of the subject (TB-25) (bottom group) in solving math problems in the aspect of CK.

The image shows handwritten mathematical work on a grey background. It starts with a fraction $\frac{60}{360}$ followed by an equals sign and the expression $\frac{60}{360} \times \pi \times r \times r$. Below this, it simplifies to $= \frac{1}{6} \times \pi \times 30 \times 30$, and finally to $= 150\pi$.

FIGURE 7. Answers to CK problems (TB-25)

The follow-up response to the subject's answer was quite short and inaccurate (**FIGURE 7**). Therefore the researcher (R) needs to conduct interviews with lower group subjects (TB-25) to dig deeper into information about CK which is applied in solving problems.

7th Interview

- R : Do you understand the concept of circle area?
 TB-25 : I really understand that concept.
 R : Do you understand the problem in test number 2?
 TB-25 : Sorry, my focus is on the circle area of the circle 60°
 R : What do you think about the results of this work?
 TB-25 : My job is not final. I should have calculated the area of the full circle and then reduced the area of 60°

DISCUSSION

In general, the level of MD of elementary school teachers is in the moderate or good enough criteria, with an average score of 72.77 (see **TABLE 4**). There 80% of teachers have a medium to high-level MD. Teachers with a high level of MD considerably effort to learn mathematics. However, 20% of teachers are still at a low level of MD. The results of interviews (1st Interview) with lower group teachers stated that mathematics was not their area of expertise. However, they are highly motivated to improve their abilities and adjust the practice of learning mathematics in the classroom (Damrongpanit, 2019).

MK data from 30 teachers achieved an average score of 71.73 with the criteria of "medium" or good enough (see **TABLE 3**). The average value (MK) combines the PK value of 71.33 and CK of 71.67. Teachers in the upper group managed to achieve an MK score of 85.83 (high). However, in the lower group (16.6%) the teacher's MK level is still relatively low with an average score (54.50) with PK 53.00 and CK 56.00. Of course, teachers who are in the lower class need to improve their competence in mastering MK more seriously.

By paying attention to the data in **TABLE 3** and **TABLE 4**, it can be stated that the group of subjects (teachers) with a high MD level also has a high MK test result. Meanwhile, subjects with lower MD levels had lower MK test results. This phenomenon was then investigated through statistical tests to determine the effect of MD on MK.

Statistical tests with SPSS 23.0 show a significant correlation and influence (Sig. $0.000 < 0.05$) between mathematics disposition and elementary school teacher mathematics knowledge. Considering the output coefficients (**TABLE 5**) a linear regression model equation can be drawn up $\hat{Y} = 11.692 + 0.825X$. The regression coefficient number is 0.825 (positive), which means that for every 1% increase in the level of mathematical disposition (X), then mathematical knowledge (Y) will increase by 0.825. The results of the data analysis then obtained the R Square value (coefficient of determination) of 0.523. It means that 52.3% of the mathematical knowledge variable (Y) variations can be explained by the mathematical disposition variable (X) through the linear regression model, while other variables influence 47.7%.

Empirical evidence and statistical tests also show a correlation between mathematical disposition and improvement in mathematical skills (procedural and conceptual knowledge). Referring to the results of previous research, it is stated that mathematical disposition is very important for teachers to support how processes, actions, and motivation improve problem-solving abilities (Hutajulu, 2019). Mathematical disposition can also increase mathematical power so that self-confidence increases when learning mathematics (Feldhaus, 2014; Hutajulu, et al., 2019; Kusmaryono, et al., 2019).

Top group PK-CK analysis

Responses to the answer to test question number 1 in the PK and CK aspect were based on an analysis of the results of the work (**FIGURE 2** and **FIGURE 5**) and interviews (2nd Interview). The results showed that the subject (TT-22) had used an appropriate completion procedure and was easy for students to implement. Subject (TT-22) in applying CK involves the relationship between one concept and another according to the problems that must be grounded. Subject (TT-22) has PK which is able to argue why the procedure is used and give the correct answer in the context of the problem described (Otun & Olaoye, 2019).

Middle group PK-CK analysis

Analysis of the response to the answer to test number 1 aspects of PK in **FIGURE 3** and the results of the interview (3rd Interview), shows that the subject (TT-16) is able to argue why the procedure is used and provides the correct answer in the context of the problem

described (Otun & Olaoye, 2019). The subject answer (TM-16) leads to understanding mathematical concepts. Subjects (TM-16) combined the application of procedural and conceptual knowledge. Good initial PK of the subject (TM-16) also shows a justifiable conceptual knowledge. Subject (TM-16) states that knowledge in problem-solving is obtained from learning experiences outside the classroom. This knowledge influences the teachers' actions in the classroom when it involves students learning mathematics (Damrongpanit, 2019).

Analysis of the response to the answer to the test number 2 aspects of CK in **FIGURE 6** and the analysis of the results of the interview (6th Interview), the subject (TM-11) has strong CK. The subject (TM-11) has the conceptual ability deductively at the symbolization stage, namely the subject formulates a representation of each concept he learns using mathematical symbols or other appropriate verbal formulations (Anwar & Rahmawati, 2017).

Mastery of PK and CK by the subject (TM-16 and TM-11) which is applied in problem-solving seems to be interrelated and two-way in nature, namely when the subject solves their procedural problems using concepts that have been mastered. This leads to an expert opinion which states that a person's PK may be supported by their CK, and or vice versa (Kadijevich, 2018; Rittle-Johnson et al., 2016).

Lower group PK-CK analysis

The response to the answer to test number 1 for aspects of PK in **FIGURE 4** shows that does not support the subject's conceptual knowledge (TB-25), but only uses intuition in making decisions (Dörfler & Ackermann, 2012). Subjects (TB-25), when asked about their solution ideas (4th Interview), did not know the principles or rules that underlie the procedures used (Kadijevich, 2018). Although, basically, the conceptual answer of the subject in problem-solving can be justified. While the response to the answer to test number 2 aspects of CK in **FIGURE 7**, the subject (TB-25) only understands a few basic concepts of circle area (7th Interview). However, the subject is wrong in applying his idea, so the answer does not match the problem to be solved.

The relationship between procedural and conceptual knowledge

In solving mathematical problems, conceptual knowledge and procedural knowledge are needed. Conceptual knowledge that is not supported by procedural knowledge will result in the teacher only having an intuition about a concept but being unable to solve a problem. On the other hand, procedural knowledge that is not supported by conceptual knowledge will result in the teacher being adept at manipulating symbols but not understanding and knowing the meaning of these symbols. This condition allows teachers to provide answers to a problem (problem) without understanding what they are doing (Kadijevich, 2018; Otun & Olaoye, 2019; Rittle-Johnson et al., 2016). There is no definite learning sequence, whether learning to master procedural knowledge at the beginning is better than learning conceptual knowledge later or vice versa. Generally, primary school teachers prefer to teach procedures first because conceptual knowledge will develop based on previous procedural knowledge. However, there is evidence that the effect of conceptual knowledge on procedural knowledge may be stronger than the other way around (Rittle-Johnson et al., 2016).

It is clear that the two mathematical knowledge, namely conceptual knowledge on procedural knowledge, are interrelated, influence each other, and are important for a teacher (Manfreda Kolar & Hodnik, 2021). As happened to subject knowledge (TT-22, TT-11, TM-16) and supported by expert opinion, that the procedural knowledge and conceptual knowledge of a person who is learning (mathematics) develops repeatedly and in two directions. Increasing one of this knowledge will affect the increase in other knowledge (Kadijevich, 2018). In other words, one kind of knowledge is based on another,

and vice versa (Jazuli et al., 2017). On the other hand, the expert opinion states that if conceptual knowledge and procedural knowledge are not interrelated then one of two possibilities will occur, namely (1) the teacher has a good intuitive understanding of mathematics but cannot solve the problem, or (2) the teacher can give the answer correctly but did not understand what they were doing (Kadijevich, 2018; Rittle-Johnson et al., 2016).

Paying attention to this discussion, we agree that teacher professional development must always be carried out because it can affect skills, knowledge, and self-confidence in carrying out educational tasks (Sasson et al., 2020). Conceptual and procedural knowledge are important aspects of understanding mathematics (Haji et al., 2019; Rittle-Johnson & Alibali, 1999). Therefore, teachers in teaching mathematics must apply both knowledges. If procedural knowledge is learned without understanding concepts, what we learn will be isolated from the knowledge. Learning new topics will experience obstacles because there is no concept network with previously learned knowledge.

The relationship between mathematical disposition, procedural knowledge, and conceptual knowledge

The results of data analysis (**TABLE 4**) show that mathematical dispositions are associated with mathematical knowledge (procedural and conceptual knowledge). However, two subjects have the same mathematical knowledge, but have different mathematical dispositions, it is strongly suspected that the two subjects will have different abilities or competencies (Hutajulu et al., 2019).

The results of data analysis in **TABLE 3** and **TABLE 4** show that by comparing the average mathematical knowledge and mathematical disposition of teachers between groups it can be seen that teachers who have higher mathematical dispositions tend to have higher mathematical knowledge (procedural and conceptual knowledge) than teachers with lower mathematical dispositions. Meanwhile, based on the interview data, it is found that subjects with high dispositions will be more persistent, diligent, interested, and challenged to explore their mathematical abilities or knowledge to find solutions to problem-solving. This makes the subject have more mathematical knowledge than subjects who do not exhibit this behavior. Furthermore, a positive mathematical disposition will have implications for the formation of mathematical competence (procedural and conceptual knowledge) of a teacher in problem-solving learning. Thus, it is clear that mathematical disposition is a prerequisite that supports the formation of mathematical knowledge (procedural and conceptual knowledge).

CONCLUSION

Based on the findings and discussion, it was concluded that there was a significant influence on the level of mathematical disposition on mathematics knowledge of elementary school teachers in solving problems on aspects of procedural and conceptual knowledge. Teachers with a high level of disposition have high procedural and conceptual knowledge as well. Meanwhile, teachers with lower disposition levels have lower procedural and conceptual knowledge.

The relationship between mathematical disposition, procedural knowledge, and conceptual knowledge, namely a positive mathematical disposition will have implications for the formation of mathematical knowledge (procedural and conceptual knowledge) of a teacher in problem-solving learning and mathematical disposition is a prerequisite that supports the formation of mathematical knowledge (procedural and conceptual knowledge).

This shows that however, mathematical disposition strongly supports the development of mathematical knowledge, especially procedural and conceptual knowledge. Therefore, it is important to suggest to educators that developing

mathematical knowledge cannot be separated from developing mathematical dispositions.

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REFERENCES

1. Ahmad Al-Khateeb, M. (2016). The extent of mathematics teacher's awareness of their students' misconceptions in learning geometrical concepts in the intermediate education stage. *European Scientific Journal, ESJ*, 12(31), 357. <https://doi.org/10.19044/esj.2016.v12n31p357>
2. Aliustaoğlu, F., Tuna, A., & Biber, A. Ç. (2018). Misconceptions of sixth grade secondary school students on fractions. *International Electronic Journal of Elementary Education*, 10(5), 591-599. <https://doi.org/10.26822/iejee.2018541308>
3. Anku, S. E. (1996). Fostering students' disposition towards mathematics: A case from a canadian university. *Education*, 116(4), 532-528. <https://bit.ly/3BCFuyQ>
4. Blazar, D., & Kraft, M. A. (2017). Teacher and Teaching Effects on Students' Attitudes and Behaviors. In *Educational Evaluation and Policy Analysis* (Vol. 39, Issue 1). <https://doi.org/10.3102/0162373716670260>
5. Boonen, A. J. H., de Koning, B. B., Jolles, J., & van der Schoot, M. (2016). Word problem solving in contemporary math education: a plea for reading comprehension skills training. *Frontiers in Psychology*, 7(February), 1-10. <https://doi.org/10.3389/fpsyg.2016.0019>
6. Castrol, F. G., Kellison, J. G., Boyd1, S. J., & Kopak, A. (2010). A Methodology for Conducting Integrative Mixed Methods Research and Data Analyses. *Journal Mix Methods Research*, 4(4), 342-360. <https://doi.org/10.1177/1558689810382916.A>
7. Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches*. SAGE Publications.
8. Damrongpanit, S. (2019). From modern teaching to mathematics achievement: The mediating role of mathematics attitude, achievement motivation, and self-efficacy. *European Journal of Educational Research*, 8(3), 713-727. <https://doi.org/10.12973/eu-jer.8.3.713>
9. Feldhaus, C. A. (2014). How Pre Service Elementary School Teachers' Mathematical Dispositions are Influenced by School Mathematics. *American International Journal of Contemporary Research*, 4(6), 91-97. http://www.aijcrnet.com/journals/Vol_4_No_6_June_2014/11.pdf
10. Gronseth, S., Brush, T., Ottenbreit-Leftwich, A., Strycker, J., Abaci, S., Easterling, W., Roman, T., Shin, S., & Leusen, P. van. (2010). Equipping the next generation of teachers: Technology preparation and practice. *Journal of Digital Learning in Teacher Education*, 27(1), 30-36. <https://doi.org/10.1080/21532974.2010.10784654>
11. Haji, S., Yumiati, Y., & Zamzaili, Z. (2019). Improving students' productive disposition through realistic mathematics education with outdoor approach. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 4(2), 101-111. <https://doi.org/10.23917/jramathedu.v4i2.8385>
12. Hutajulu, M., Wijaya, T. T., & Hidayat, W. (2019). the Effect of mathematical disposition and learning motivation on problem solving: An analysis. *Infinity Journal*, 8(2), 229. <https://doi.org/10.22460/infinity.v8i2.p229-238>
13. Isaac, O. E., & Chikweru, A. E. (2018). Test for significance of Pearson's correlation coefficient (r). *International Journal of Innovative Mathematics, Statistics & Energy Policies*, 1(1), 11-23. <https://bit.ly/3zrw6eM>

14. Jazuli, A., Setyosari, P., Sulthon, & Kuswandi, D. (2017). Improving conceptual understanding and problem-solving in mathematics through a contextual learning strategy. *Global Journal of Engineering Education*, 19(1), 49–53. <https://bit.ly/3oNZDun>
15. Johnson, R. B., & Onwuegbuzie, A. J. (2007). Toward a definition of mixed methods research. *Journal of Mixed Methods Research*, 1(2), 112–133. <https://doi.org/10.1177/1558689806298224>
16. Kim, S., Raza, M., & Seidman, E. (2019). Improving 21st-century teaching skills: The key to effective 21st-century learners. *Research in Comparative and International Education*, 14(1), 99–117. <https://doi.org/10.1177/1745499919829214>
17. Kroll, T., & Neri, M. (2009). *Designs for Mixed Method Research*. Wiley-Blackwell, Hoboken.
18. Kusmaryono, I., Basir, M. A., & Saputro, B. A. (2020). Ontological misconception in mathematics teaching in elementary schools. *Infinity Journal*, 9(1), 15. <https://doi.org/10.22460/infinity.v9i1.p15-30>
19. Kusmaryono, I., Suyitno, H., Dwijanto, D., & Dwidayati, N. (2019). The effect of mathematical disposition on mathematical power formation: Review of dispositional mental functions. *International Journal of Instruction*, 12(1), 343–356. <https://doi.org/https://doi.org/10.29333/iji.2019.12123a>
20. Manfreda, V. (2021). Mathematical Literacy from the Perspective of Solving Contextual Problems. *European Journal of Educational Research*, 10(1), 467–483. <https://doi.org/10.12973/eu-jer.10.1.467>
21. Markovits, Z., & Patkin, D. (2020). Preschool in-service teachers and geometry: attitudes, beliefs and knowledge. *International Electronic Journal of Mathematics Education*, 16(1), em0619. <https://doi.org/10.29333/iejme/9303>.
22. Miles, M. B., & Huberman, M. A. (2016). Qualitative data analysis: A resource book on new methods. In *Universitas Indonesia UI Press* (11th ed., Issue 1). Universitas Indonesia (UI-Press).
23. Mulyono, Rahmawati, M. I., & Amidi. (2019). The ability of mathematical problem solving reviewed from goal orientation to learning model of problem based learning assisted by problem card. *Unnes Journal of Mathematics Education*, 8(1), 8–13. <https://journal.unnes.ac.id/sju/index.php/ujme/article/view/29134>
24. Ojose, B. (2015). Students' Misconceptions in Mathematics: Analysis of Remedies and What Research Says. *Ohio Journal of School Mathematics*, 72(1), 30–34. <https://bit.ly/3Bzcliz>.
25. Österman, T., & Bråting, K. (2019). Dewey and mathematical practice: revisiting the distinction between procedural and conceptual knowledge. *Journal of Curriculum Studies*, 51(4), 457–470. <https://doi.org/10.1080/00220272.2019.1594388>
26. Otun, W. I., & Olaoye, A. A. (2019). Enhancing the conceptual, procedural and flexible procedural knowledge of pre-service mathematics teachers in algebra. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 4(2), 66–78. <https://doi.org/10.23917/jramathedu.v4i2.8363>
27. Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, 91(1), 175–189. <https://doi.org/10.1037/0022-0663.91.1.175>
28. Rittle-Johnson, B., Fyfe, E. R., & Loehr, A. M. (2016). Improving conceptual and procedural knowledge: The impact of instructional content within a mathematics lesson. *British Journal of Educational Psychology*, 86(4), 576–591. <https://doi.org/10.1111/bjep.12124>
29. Salisu, A., & Ransom, E. N. (2014). The role of modeling towards impacting quality education. *International Letters of Social and Humanistic Sciences*, 32(2), 54–61. <https://doi.org/10.18052/www.scipress.com/ilshs.32.54>

30. Sasson, I., Kalir, D., & Malkinson, N. (2020). The role of pedagogical practices in novice teachers' work. *European Journal of Educational Research*, 9(2), 457–469. <https://doi.org/10.12973/eu-jer.9.2.457>
31. Taber, K. S. (2018). The use of Cronbach's Alpha when developing and reporting research instruments in science education. *Research in Science Education*, 48(6), 1273–1296. <https://doi.org/10.1007/s11165-016-9602-2>

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