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Uncertainty of Helium Ion Momentum (He^+) by Using the Heisenberg Uncertainty Approach on Quantum Numbers $n \leq 3$

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Abstract

This study aims to find the uncertainty value of Helium Ion Momentum (He^+) by using the Heisenberg Uncertainty Approach to Quantum Numbers $n \leq 3$. This type of research uses non-experimental research. The research was conducted by developing previously existing theories. This research is to calculate the expectation value of the position to find out how often electrons can appear, and determine the uncertainty of the momentum of Helium Ion (He^+) by using the Heisenberg uncertainty approach at number $n \leq 3$. Based on the results and discussion, it can be concluded that the uncertainty of helium ion momentum (He^+) by using the Heisenberg uncertainty approach to quantum numbers $n \leq 3$. The uncertainty of helium ion momentum is influenced by the main quantum number (n) and the azimuthal quantum number (l). By entering the values of n and l, the results can be obtained at quantum numbers $n = 1$ with $l = 0$ obtained $\Delta P_r = 2{,}977504663 \times 10^{-24}$. At the quantum number $n = 2$ with $l = 0$ obtained $\Delta P_r = 8{,}06312759 \times 10^{-24}$ and $l = 1$ obtained $\Delta P_r = 2{,}03048 \times 10^{-23}$. At the quantum number $n = 3$ with $l = 0$ obtained $\Delta P_r = 1{,}50366086 \times 10^{-23}$, and $l = 1$ obtained $\Delta P_r = 3{,}481407588 \times 10^{-23}$ and $l = 2$ obtained $\Delta P_r = 4{,}47363642 \times 10^{-22}$. Based on these results, it can be concluded that the uncertainty value of momentum is getting bigger as the two values of n and l increase. This is also proven by using simulation and the results of momentum uncertainty are not much different and only a difference of a few decimal places.
Keywords: Heisenberg, momentum uncertainty, quantum, Schrodinger

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INTRODUCTION

In the universe, many types of elements have been identified. Some of them are reactive, while others are not. Elements that tend to be non-reactive can be found in Group VIIIA of the periodic table. Helium atoms are one such example. These atoms are known for their high stability, making them very difficult to react with other elements. The atomic structure of helium is relatively simple, with only two electrons orbiting the nucleus (Hanafi et al., 2016).

The helium atom consists of a nucleus composed of two protons and two neutrons, surrounded by a pair of electrons. In the context of quantum mechanics, the description and behavior of the particles constituting this atom are understood through the principles of quantum mechanics. Electrons, from the perspective of quantum mechanics, are considered as waves whose properties are described by wave functions, which are solutions to the Schrödinger equation. When considering two-particle systems such as the helium atom, the solution to the Schrödinger equation involves the multiplication of two wave functions, each describing the involved particle (Krishna et al., 2018).

Helium ions (He^+) are commonly found in hot stars such as the sun. The formation process of these ions occurs when fast-moving helium atoms in the sun's extremely hot atmosphere collide with other atoms, causing the release of electrons and forming positively charged helium ions (He^+). Due to this electron loss, the helium ion (He^+) has only one electron, thus it can be considered to have "hydrogenic" properties (Damayanti et al., 2019). The concept of a hydrogenic atom refers to an atom that has lost electrons, leaving one electron in its outer orbit. Therefore, the wave function for the helium ion (He^+) can be analyzed using the Schrödinger equation, which is typically used for the hydrogen atom, which has one electron (Gautreau and Savin, 2006:98). The radial function of the helium ion at principal quantum numbers (n) ≤ 3 can be seen in Table 1

Table 1. Radial function of helium ion at principal quantum numbers *(n) ≤ 3*

n	l	$R_{nl}(r)$
1	0	$R_{10}(r) = \frac{2}{(a_0)^{\frac{3}{2}}} e^{-\frac{r}{a_0}}$
2	0	$R_{20}(r) = \frac{\left(2-\left(\frac{r}{a_0}\right)\right)}{2\sqrt{2a_0^{3}}} e^{-\frac{r}{2a_0}}$
	1	$R_{21}(r) = \frac{1}{\sqrt{3}(2a_0)^{\frac{3}{2}}} \left(\frac{r}{a_0}\right) e^{-\frac{r}{2a_0}}$
3	0	$R_{30}(r) = \frac{2}{81\sqrt{3}a_0^{\frac{3}{2}}} \left[27 - \frac{18r}{a_0} + \frac{2r^2}{a_0^2}\right] e^{-\frac{r}{3a_0}}$
	1	$R_{31}(r) = \frac{4}{81\sqrt{6a_0}} \left(\frac{6r}{a_0} - \frac{r^2}{a_0^2}\right) e^{-\frac{r}{3a_0}}$
	2	$R_{32}(r) = \frac{4}{27\sqrt{10}(3a_0)^{\frac{3}{2}}} \left(\frac{r^2}{a_0^2}\right) e^{-\frac{r}{3a_0}}$

(Makmun *et al.,* 2020).

Helium has various important applications in physics, particularly in research and experiments. One of its primary uses is in Helium-Neon (HeNe) lasers, where certain studies measure the speed of HeNe laser beams propagating through air and optical fiber using a 200 MHz oscilloscope. The research results indicate that the speed of HeNe laser beams in the air is approximately (2.87±0.12)×10^8 m/s, whereas in optical fiber it is about (1.84±0.08)×10^8 m/s (Andelita et al., 2021). Additionally, helium is used in the purification systems of small-scale High-Temperature Gas-cooled Reactors (HTGR), which are part of the development of nuclear energy technology (Nurroniah et al., 2023).

In quantum physics, uncertainty is an intrinsic property of the physical quantities obtained in measurement processes. The Heisenberg uncertainty principle arises from the wave-particle duality, the notion that every particle can exhibit both wave-like and particle-like properties. This concept was first introduced by Louis de Broglie, who posited that particles such as electrons have a wavelength associated with their momentum through the equation λ=h/p. De Broglie hypothesized that material particles could behave like waves, meaning they have a specific wavelength related to their momentum.

a specific wavelength associated with its momentum. This hypothesis combines the classical properties of particles with the properties of waves, showing that on a quantum scale, particles cannot be described simply as entities that have fixed position and momentum, but rather as entities that have probabilistic properties described by wave functions. This principle is the basis of the Heisenberg uncertainty principle, which states that the uncertainty in the measurement of particle position and momentum cannot be eliminated, but always has a minimal limit determined by Planck's constant. (Nurlina, 2017: 72)

Wave function ψ is a complex mathematical entity that contains all the information that might be known about a quantum system. To give this wave function a physical meaning, we use the probabilistic interpretation first introduced by Max Born. According to Born's interpretation, the squared absolute value of the wave function |ψ(x)|^2represents the probability density of finding a particle at any position in space. x in space. This means that if we take many measurements on an identical system, the distribution of particle position measurements will follow the pattern defined by |ψ(x)|^2. When we measure the particle's position very accurately (so that the uncertainty of the position ∆p becomes very small), the wave function becomes very narrow in position space, indicating that the particle is more localized. However, due to the Fourier transform between position space and momentum space, the narrowing of the wave function in position space causes the wave function in momentum space to become wider. As a result, the momentum uncertainty ∆p increases. This reflects the Heisenberg uncertainty principle, which states that the product of uncertainties in position and momentum cannot be smaller than a certain limit set by Planck's constant. So, the more accurately we measure the position of a particle, the greater the uncertainty in its momentum, and vice versa (Daniaty, 2014: 126).

The Heisenberg uncertainty principle is expressed mathematically with the equation:

$$\Delta x.\Delta p \geq \frac{\hbar}{2} \quad (1)$$

This principle states that the more accurately we measure the position of a particle (the smaller ∆x, the greater the uncertainty in the measurement of its momentum (the larger ∆p, and vice versa.) This is not a result of the limitations of measuring devices, but rather the fundamental nature of quantum particles. This is not a result of the limitations of measuring devices, but rather a fundamental property of quantum particles. This suggests that the product of the uncertainties in position and momentum cannot be smaller than a certain limit determined by Planck's constant. It is this wave-particle duality and the nature of the wave function that explains why there are fundamental limits in the precision of simultaneous measurements of these variables. This uncertainty principle reflects the probabilistic and non-deterministic nature of the quantum world, where particles do not have a definite position and momentum before being measured, but only a probability distribution of existence given by the wave function. This principle also shows that at the quantum level, there is a natural limit to knowing these two quantities simultaneously with infinite precision (Bagus et al, 2019).

METHODS

This type of research uses non-experimental methods. The research was conducted by developing previously existing theories. This research aims to calculate the expectation value of the position to find out how often electrons can appear and determine the uncertainty of the momentum of the Helium Ion (He^+) using the Heisenberg uncertainty approach at quantum number n≤3.

The certainty of the position of an electron can be determined from the expectation value, namely by using the wave function (ψ) with the assumption that the electron exists in three dimensions so that the expectation value equation is:

$$\langle r\rangle = \int_{-\infty}^{\infty} r^3 R^2 dr \tag{2}$$

Next, find the probability, with the expectation value and the squared expectation value of the Helium ion for various states against the position. This will help in formulating the Heisenberg uncertainty on helium ions (He^+) specifically the squared expectation price:

$$\langle r^2\rangle = \int_{-\infty}^{\infty} r^4 R^2 dr \tag{3}$$

So that a radial position uncertainty price is obtained, with:

$$\Delta r = \sqrt{\langle r^2\rangle - \langle r\rangle^2} \tag{4}$$

Then for the magnitude of the uncertainty of the radial momentum of the Helium Ion can use the Heisenberg uncertainty approach:

$$\Delta P_r \geq \frac{\hbar}{2}\Delta r \tag{5}$$

There are several steps in this research, namely the preparation stage, theory development, data collection, discussion, and conclusions. At the initial stage, namely the preparation stage, researchers collected some information from existing literature. Researchers collected information from several previous articles, quantum physics books, and other physics books, as well as other information from the internet. In the second stage, namely theory development, researchers developed theories from pre-existing theories.

The next stage is data collection with calculations using the equations that have been written previously. Furthermore, validation of the results obtained from the calculation is carried out, the results obtained are matched with simulations using Matlab and literature or previous articles that have been done before. Testing the results of momentum uncertainty using Matlab R2015a software which is carried out with the following steps: First, create a position expectation m-file by applying the radial function (Rnl) from the literature presented in the table 1 The following is an image of the expectation of r,

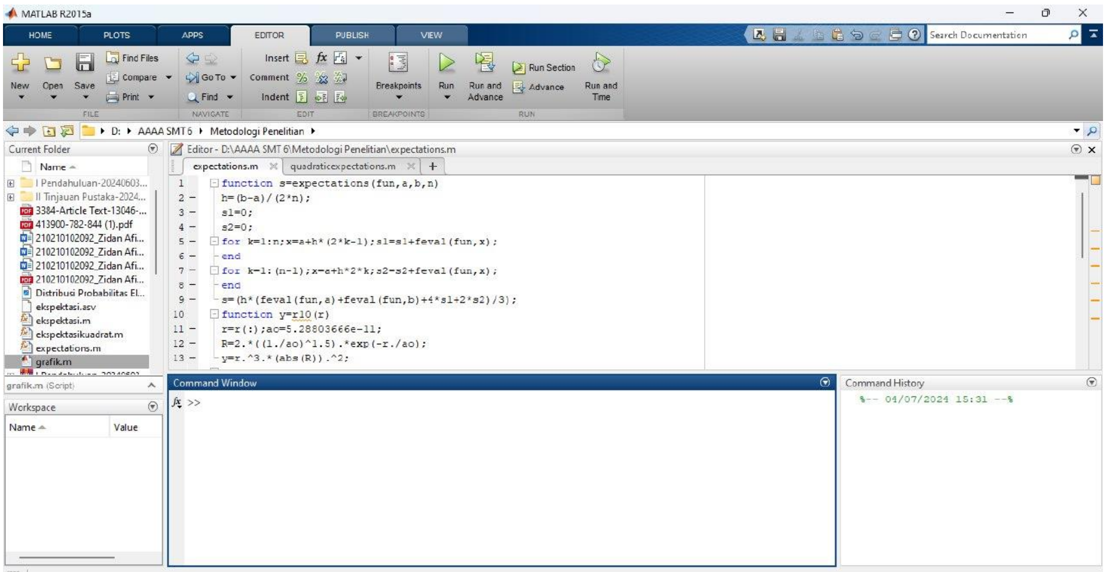

Figure 1. Expectation r

Then create an m-file of the squared expectation of r. Here's an image of the squared expectation of r

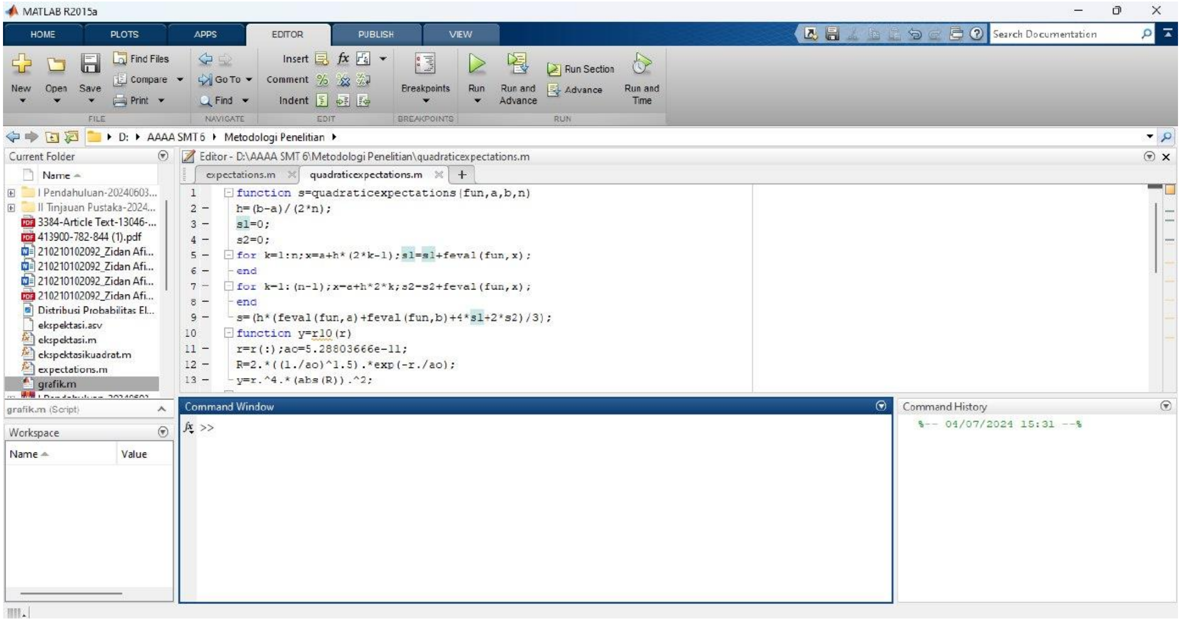

Figure 2. Squared Expectation r

The next step is to write a script in the command window to determine the expected position $\langle r\rangle$, squared expectation position $\langle r^2\rangle$, uncertainty position (Δr), and momentum uncertainty (Δp). The scripts to determine these values are shown in Figures 3 and Figure 4.

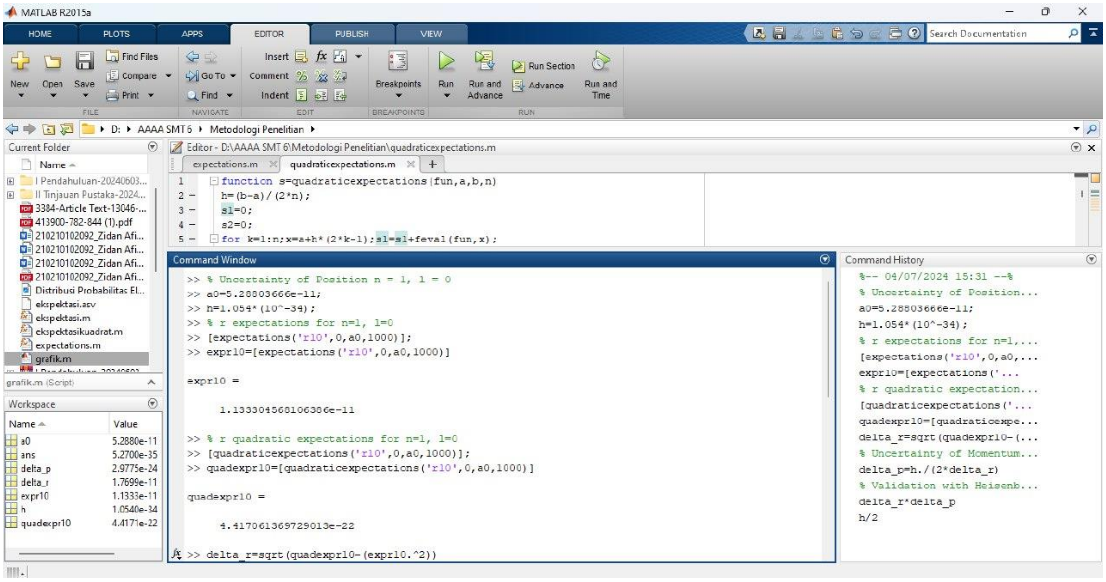

Figure 3. *The script process of position expectation* $\langle r\rangle$, *position squared expectation* $\langle r^2\rangle$, *position uncertainty* (Δr), *and momentum uncertainty* (Δp)

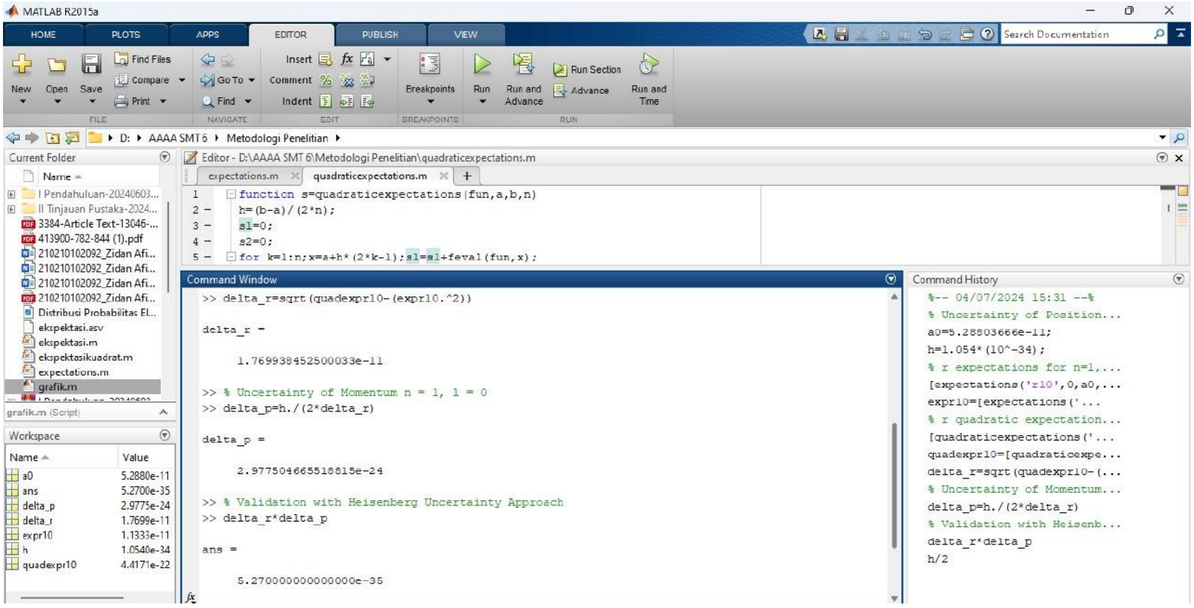

Figure 4. The script process of position expectation $\langle r \rangle$, position squared expectation $\langle r^2 \rangle$, position uncertainty (Δr), and momentum uncertainty (Δp)

Then do the same for the variation of quantum number n≤3 and display the momentum uncertainty graph according to the value obtained.

After that is the discussion stage where it is explained about solving the Helium Ion wave function problem (He^+) in position space. Researchers explain through data analysis of the research results using methods, techniques, and theoretical foundations that have been chosen before. The discussion presents data and information that has been found from the results of the research and can then be used to conclude the research that has been done. Then in the last stage, namely the stage of drawing conclusions where researchers conclude the results that have been obtained concisely and clearly (Damayanti *et al,* 2019).

RESULTS AND DISCUSSION

Based on the literature, various provisions were determined, namely the plack constant ($\hbar = h/2\pi = 1{,}0546 \times 10^{-34}$), proton mass ($m_p = 1{,}6726 \times 10^{-27}$), mass electron ($m_e = 9{,}1094 \times 10^{-31}$), and mass neutron ($m_n = 1{,}675 \times 10^{-27}$). The equation for the Bohr radius of the Hydrogen atom:

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}n^2$$

(Krane, 2012).

Hydrogenic ions other than hydrogen have different Z values, for example Helium Ions which have Z = 2. (Supriadi *et al.*, 2023). So the mass used is the reduced mass. Therefore, the above equation becomes:

$$a_0 = \frac{n^2\hbar^2 4\pi\varepsilon_0}{e^2\mu}$$

Where μ is the reduced mass of the deutron and electron in the helium ion. Mathematically written:

$$\mu = \frac{\left(2(m_p + m_n)\right)\cdot m_e}{\left(2(m_p + m_n)\right) + m_e}$$

By solving mathematically, the value of the Bohr radius of the Helium Ion is found to be $a_0 = 0{,}528803666 \times 10^{-10}\, m$.

The wave function of helium ions can be obtained using the Schrodinger equation in spherical coordinates using separation of variables. The wave function consists of radial wave function and angular wave function. The radial wave function can present that electrons can be found at a distance of the electron orbit (r) (Makmun *et al.*, 2020). By solving the normalized Schrodinger equation of helium ions, the radial wave function is obtained as follows:

$$R_{nl}(r) = -\left(\frac{2}{na_0}\right)^{\frac{3}{2}} \sqrt{\frac{(n-l-1)!}{2n[(n+l)!]^3}} \left(\frac{2r}{na_0}\right)^l e^{-\frac{r}{na_0}} L_{n+l}^{2l+1}\left(\frac{2}{na_0}\right)$$

From the radial wave function, it is known that the radial wave function depends on the main quantum number and the azimuthal quantum number. The radial wave function can be used to determine the expected value of the electron position or the average value of the electron position measurement (Supriadi *et al.*, 2022). The position expectation value for quantum numbers $n = 1$ and l = 0 is as follows:

$$\langle r\rangle = \int_0^{a_0} r^3\, |R_{10}|^2 dr$$

$$\langle r\rangle = \int_0^{a_0} r^3 \left|\frac{2}{(a_0)^{3}/_2} e^{\frac{-r}{a_0}}\right|^2 dr$$

$$\langle r\rangle = \frac{4}{a_0^3}\int_0^{a_0} r^3\, e^{\frac{-2r}{a_0}}\, dr$$

$$\langle r\rangle = \frac{4}{a_0^3}\left[-\frac{a_0}{2} r^3 e^{\frac{-2r}{a_0}} - \frac{3a_0^2}{4} r^2 e^{\frac{-2r}{a_0}} - \frac{6a_0^3}{8} r e^{\frac{-2r}{a_0}} - \frac{6a_0^4}{16} e^{\frac{-2r}{a_0}} dr\right]\Bigg|_0^{a_0}$$

$$\langle r\rangle = \frac{4}{a_0^3}\left[\frac{(-0{,}5a_0^4 - 0{,}75a_0^4 - 0{,}75\,a_0^4 - 0{,}375\,a_0^4)}{e^2} - (-0{,}375\,a_0^4)\right]$$

$$\langle r\rangle = \frac{4}{a_0^3}\left[\frac{-2{,}375\,a_0^4}{(2{,}718281828459045)^2} + 0{,}375\,a_0^4\right]$$

$$\langle r\rangle = \frac{4}{a_0^3}\left[\frac{-2{,}375\,a_0^4}{7{,}38905609893} + 0{,}375\,a_0^4\right]$$

$$\langle r\rangle = \frac{4}{a_0^3}(0{,}0535787024a_0^4)$$

$$\langle r\rangle = 0{,}2143148096a_0$$

The expectation value of the squared position for quantum numbers $n = 1$ and $l = 0$ is as follows:

$$\langle r^2\rangle = \int_0^{a_0} r^4\, |R_{10}|^2\, dr$$

$$\langle r^2\rangle = \int_0^{a_0} r^4 \left|\frac{2}{(a_0)^{3}/_2} e^{\frac{-r}{a_0}}\right|^2 dr$$

$$\langle r^2\rangle = \left(\frac{2}{(a_0)^{3}/_2}\right)^2 \int_0^{a_0} r^4\; e^{\frac{-2r}{a_0}}\; dr$$

$$\langle r^2\rangle = \frac{4}{a_0^3}\left[-\frac{a_0}{2} r^4\, e^{\frac{-2r}{a_0}} - \frac{4a_0^2}{4} r^3 e^{\frac{-2r}{a_0}} + \frac{12\,a_0^3}{8} r^2 e^{\frac{-2r}{a_0}} + \frac{24\,a_0^4}{16} r\, e^{\frac{-2r}{a_0}} + \frac{24\,a_0^5}{32} e^{\frac{-2r}{a_0}}\right]$$

$$\langle r^2\rangle = \frac{4}{a_0^3}\left[\frac{(-0{,}5a_0^5 - a_0^5 - 1{,}5a_0^5 - 1{,}5a_0^5 - 0{,}75\,a_0^5)}{e^2} - (-0{,}75a_0^5)\right]$$

$$\langle r^2\rangle = \frac{4}{a_0^3}\left[\frac{(-5{,}25\,a_0^5)}{7{,}38905609893} + 0{,}75a_0^5\right]$$

$$\langle r^2\rangle = \frac{4}{a_0^3}\,[-0{,}7105102369a_0^5 + 0{,}75a_0^5]$$

$$\langle r^2\rangle = \frac{4}{a_0^3}(0{,}039489763a_0^5)$$

$$\langle r^2\rangle = 0{,}1579590524a_0^2$$

In the same way, the expected value of the position of the helium ion electron at the main quantum number $(n) \leq 3$ in table 2 is obtained.

Table 2. Calculation results of position expectation values and square position expectation of helium ion electrons at the main quantum number $(n) \leq 3$

n	l	$\langle r \rangle$	$\langle r^2 \rangle$
1	0	$0{,}2143148096\ a_0$	$0{,}1579590524\ a_0^2$
2	0	$0{,}02959075\ a_0$	$0{,}01575875\ a_0^2$
	1	$0{,}00297091667\ a_0$	$0{,}00249725\ a_0^2$
3	0	$0{,}0090868092\ a_0$	$0{,}00447528\ a_0^2$
	1	$0{,}001020263171\ a_0$	$0{,}000961034\ a_0^2$
	2	$0{,}0000056304126\ a_0$	$0{,}000004962650815\ a_0^2$

In this study, it was found that the greater the main quantum number (n), the smaller the expected value of the position of electrons found in helium ions. This is in accordance with research by Bawani et al. (2023) which states that the relationship between the magnitude of the quantum number and the expected value of the electron position is inversely proportional, this is because if the electron is at a large main quantum number, the average distance of the electron to the atomic nucleus will be further away, and the electron will be found less often. The position expectation value and the squared position expectation value are then used to determine the uncertainty value of the electron position in the helium ion. According to Heisenberg Uncertainty the behavior of matter and waves cannot be determined simultaneously. The principle of uncertainty states that position and momentum cannot be expressed simultaneously. This is because the smaller the position uncertainty value will cause the momentum uncertainty value to be more inaccurate, and vice versa (Irfan et al., 2023). The uncertainty value of the electron position for quantum numbers $n = 1$ and $l = 0$ is as follows:

$$\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2}$$

$$\Delta r = \sqrt{(0{,}1579590524\ a_0{}^2) - (0{,}2143148096\ a_0)^2}$$

$$\Delta r = \sqrt{(0{,}1579590524\ a_0{}^2) - (0{,}0459308376\ a_0{}^2)}$$

$$\Delta r = \sqrt{0{,}1120282148\ a_0{}^2}$$

$$\Delta r = 0{,}3347061619\ a_0$$

$$\Delta r = 0{,}1769938454 \times 10^{-10}\ m$$

In a similar manner, Table 3 shows the uncertainty values of the position of the helium ion electron at principal quantum numbers $(n) \leq 3$.

Table 3. *Calculation results of the uncertainty values of the position of the helium ion electron at principal quantum numbers* $(n) \leq 3$

n	l	Δr
1	0	$0{,}1769938454 \times 10^{-10}\ m$
2	0	$0{,}0653592537 \times 10^{-10}\ m$
	1	$0{,}0259544388 \times 10^{-10}\ m$
3	0	$0{,}0350477965 \times 10^{-10}\ m$
	1	$0{,}01607249137 \times 10^{-10}\ m$
	2	$0{,}001178012582 \times 10^{-10}\ m$

The obtained uncertainty values of the electron's position are used to determine the momentum uncertainty of the helium ion electron at the principal quantum number n $\leq$ 3. The electron's momentum uncertainty values for the quantum numbers $n = 1$ dan $l = 0$ are as follows:

$$\Delta P_r \geq \frac{\hbar}{2\Delta r}$$
$$\Delta P_r \geq \frac{1{,}054 \times 10^{-34} J.s}{2\,(0{,}1769938454 \times 10^{-10}\, m)}$$
$$\Delta P_r \geq \frac{1{,}054 \times 10^{-34} J.s}{0{,}3539876908 \times 10^{-10}\, m}$$
$$\Delta P_r \geq 2{,}977504663 \times 10^{-24}\, kgm/s$$

In a similar manner, Table 4 shows the uncertainty values of the momentum of the helium ion electron at principal quantum numbers $(n) \leq 3$.

Table 4. *Calculation results of the uncertainty values of the momentum of the helium ion electron at principal quantum numbers n ≤ 3*

n	l	ΔP_r
1	0	$2{,}977504663 \times 10^{-24}$
2	0	$8{,}06312759 \times 10^{-24}$
	1	$2{,}03048 \times 10^{-23}$
3	0	$1{,}50366086 \times 10^{-23}$
	1	$3{,}481407588 \times 10^{-23}$
	2	$4{,}47363642 \times 10^{-22}$

To validate the results of the position uncertainty and momentum uncertainty, proof was conducted using Matlab simulations. Table 5 shows the results obtained from the Matlab simulations.

Table 5. *Simulation results of the position uncertainty and momentum uncertainty of the helium ion electron at principal quantum numbers* $(n) \leq 3$

n	l	Δr	ΔP_r
1	0	$0{,}176993845250003 \times 10^{-10}\, m$	$2{,}97750466551881 \times 10^{-24}$
2	0	$0{,}06535930968544 \times 10^{-10}\, m$	$8{,}06312065620553 \times 10^{-24}$
	1	$0{,}02637881362627 \times 10^{-10}\, m$	$1{,}99781539634929 \times 10^{-23}$
3	0	$0{,}035219955190053 \times 10^{-10}\, m$	$1{,}49631081912574 \times 10^{-23}$
	1	$0{,}01546835480977 \times 10^{-10}\, m$	$3{,}40695572658613 \times 10^{-23}$
	2	$0{,}00117843230212 \times 10^{-10}\, m$	$4{,}47204306138092 \times 10^{-22}$

To present the magnitude of radial momentum uncertainty at each quantum number, refer to Graph 1 below.

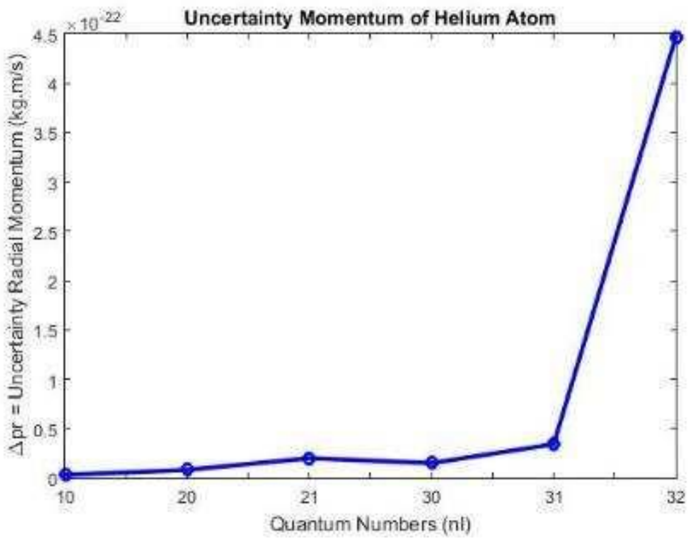

Figure 5. Graph of radial momentum uncertainty at each quantum number for the helium ion at principal quantum numbers. $(n) \leq 3$

Based on the graph, it can be explained about the relationship of radial momentum uncertainty (Δpr) which depends on the quantum number (nl). The relationship between the two is directly proportional, which means that the higher the value of n or the quantum number, the greater the value and Δpr or the uncertainty of radial momentum.

Based on the results from Table 3, Table 4, and Table 5, the same results regarding position uncertainty and momentum uncertainty were obtained between the calculations and the simulations. The explanation for these results is that none of the momentum uncertainties in position space (ΔP_r) for the state n ≤ 3 are negative (Makmun *et al.*, 2020). Figure 5 shows that the uncertainty in momentum value in position space is influenced by the principal quantum number (n) and azimuthal quantum number (l). The greater the principal quantum number (n) and azimuthal quantum number (l), the larger the uncertainty in the electron's momentum value in position space. This is because, if the electron is in a higher quantum number, the average distance between the electron and the nucleus will be greater. As the distance between the electron and the nucleus increases, the likelihood of finding the electron decreases, resulting in a larger uncertainty in the electron's momentum value in position space for helium ion.

CONCLUSION

Based on the results and discussion, it can be concluded that the uncertainty in the momentum of helium ion (He^+)n position space (ΔP_r) using the Heisenberg uncertainty principle with quantum numbers $n \leq 3$ is influenced by the principal quantum number (*n*) and azimuthal quantum number (*l*). The relationship between the uncertainty in the momentum of helium ion and the quantum numbers is direct. As the quantum numbers increase, the uncertainty in momentum also increases. This can be observed from the data of momentum uncertainty values. For quantum number $n = 1$ with $l = 0$ the uncertainty in momentum$\Delta P_r = 2{,}977504663 \times 10^{-24}$. For quantum number $n = 2$ with $l = 0$ the uncertainty in momentum$\Delta P_r = 8{,}06312759 \times 10^{-24}$ and for $l = 1$ The uncertainty in momentum$\Delta P_r = 2{,}03048 \times 10^{-23}$. For quantum number $n = 3$ with $l = 0$ the uncertainty in momentum$\Delta P_r = 1{,}50366086 \times 10^{-23}$, and $l = 1$ the uncertainty in momentum$\Delta P_r = 3{,}481407588 \times 10^{-23}$, and$l = 2$ the uncertainty in momentum$\Delta P_r = 4{,}47363642 \times 10^{-22}$. This is because as the electron occupies higher quantum numbers, the distance between the nucleus and the electron increases, resulting in a larger uncertainty in momentum.

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