Jurnal Pendidikan Fisika dan Keilmuan (JPFK)
Vol. 9, No 1. March 2023, pp. 45-52
P-ISSN: 2442-8868, E-ISSN: 2442-904X, DOI: http://doi.org/10.25273/jpfk.v9i1.16523

Radial Wave Function of Helium Ion $\left({}^{4}_{2}He^{+}\right)$ in Momentum Space at N ≤ 4

[1]Zhahrotun Nurroniah , [2]*Bambang Supriadi, [3]Vandaria Dewi Cahyani, [4]Nanda Salsabila Ayu , and [5]Kamilatul Imaniyah

[1,2] Department of Physics Education, Faculty of Teacher Training and Education, University of Jember
Jl. Kalimantan II/24 Kampus Bumi Tegal Boto, Jember 68121, Jawa Timur, Indonesia
e-mail: *zhahrotun6@gmail.com;*bambangsupriadi.fkip@unej.ac.id*

*** Corresponding Author**

Abstract

The Schrodinger equation is one of the equations in the quantum mechanical system that studies atoms, atomic nuclei, and matter in solids. The resulting Schrodinger equation can be a complex analytical solution, such as in solving problems on the wave function of helium ions. The purpose of this research is to study the wave function of helium ions in momentum space n ≤ 4. The results obtained from the wave function of helium ions in momentum space at n ≤ 4 using the Schrodinger Equation in spherical coordinates produce a complex wave function in the form of radial equations by transforming the wave function in position space using the Fourier Transform. The form of the radial wave function equation of Helium Ion is : $F_{nl} = \frac{2^{2l+\frac{5}{2}}.3^{3l+\frac{5}{2}}}{\pi^{\frac{1}{2}}} n^2 l! \left(\frac{n(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}} n^l p^l \left(\frac{p_0{}^{l+\frac{5}{2}}}{[n^2p^2+9p_0{}^2]^{l+2}}\right) C_{n-l-1}^{l+1}\left(\frac{n^2p^2-9p_0{}^2}{n^2p^2+9p_0{}^2}\right)$.

Keywords: helium ions, momentum, schrodinger equation

How to Cite: Supriadi, B., Nurroniah, Z., Cahyani, V. D., Ayu, N. D., Imaniyah, K. (2023). Radial Wave Function of Helium Ion in Momentum Space at N ≤ 4. *Jurnal Pendidikan Fisika dan Keilmuan (JPFK)*, 9(1), 45-52. doi : http://doi.org/10.25273/jpfk.v9i1.16523

Introduction

Physics is one of the natural sciences related to how to find out about natural phenomena that can be observed and measured systematically. According to Wea (2021), the emergence of quantum physics theory begins with several experiments, one of which is related to the dualism of particles and waves that cannot be explained using classical theory. In other words, particles and waves in this case cannot be distinguished when they can behave as particles or waves, so of course these two things have a very basic relationship. Quantum theory was first described by Max Planck in his scientific work in 1900. Quantum physics deals with various atomic or subatomic scale problems that study the properties of particles (Supriadi, 2022). Of course, the existence of quantum theory can expand the range of investigation in the world of physics. The difference between classical physics and quantum physics is that the study of classical physics is related to physical phenomena in the macroscopic world, while the study of quantum physics studies related to physical phenomena in the microscopic world. Where in quantum physics a particle is said to be elementary if the particle is not composed of other particles or not obtained from several other particles (Nurlina, 2017).

One of the most studied developments today is hydrogenic atoms. Hydrogen atoms are among the simplest and lightest atoms because they only contain a proton and an electron in their orbitals. Helium is one of the noble gas atoms that has 2 electrons in its orbitals, 2 protons, and 2 neutrons. If one of the electrons in the helium atom is ionized, it will become a helium ion which is characterized as a hydrogenic atom. This is as stated by Gautreau and Savin (2006) that hydrogenic atoms are single-electron atoms in their outermost orbitals. Thus, helium ions $\left({}^{4}_{2}\boldsymbol{He}^{+}\right)$ behave the same as hydrogen, the difference is that the nucleus is positively charged 2e,

where Z = 2 is the atomic number. The helium ion $({}^{4}_{2}He^{+})$ is one of the eight isotopes of the helium atom. Experimentally, helium-4 contains a total energy of -28300.7KeV (Lutfin & A, 2020).

In everyday life, helium ions have various uses including being used as a balloon filler, as a refrigerant, and as artificial air to support seabed diving (Makmun et al., 2020). In liquid form, helium is used as a chronic coolant and is commercially produced to be used as a semiconductor magnet, namely in Magnetic Resonance Imaging (MRI), Nuclear Magnet Resonance (NMR) (Rillo et al., 2015). Alimah (2016) also stated that helium can be applied as a coolant because it is characterized as an inert gas or ideal gas and is useful for heat transfer purposes. Helium ions can be found in the sun. This ion comes from the result of the rapid movement of helium atoms in the sun's high-temperature atmosphere. Helium atoms collide with other atoms to release electrons and form helium ions $({}^{4}_{2}He^{+})$. The release of electrons results in helium ions having a single electron and hydrogenic properties. Therefore, the wave function of the helium ion can be determined using the Schrodinger equation for single-electron hydrogen (Damayanti et al., 2019).

One method that can produce wave functions owned by helium ions $({}^{4}_{2}He^{+})$ is to use a solution to the Schrodinger equation (Hanafi et al., 2015). Schrodinger equation is a second-order partial differential equation that is used to provide information about the wave behavior of particles. A differential equation will produce a solution that is by quantum physics, although hindered by the absence of experimental results that can be applied as a comparison material (Krane, 1992).

According to Hermanto (2016), solving the Schrodinger equation on atoms that have a single electron can be separated into equations depending on the radius (r) and depending on the angle with the variable separation method. In addition, Fuadah (2018) also stated that the deuterium atomic wave function contains radial and angular wave functions.

In quantum mechanics, the wave function can be written in equation (1) below.

$$\psi_{(x,t)} = Ae^{-i\omega\left(t-\frac{x}{v}\right)} \quad (1)$$

If $\omega = \frac{E}{h}$ and E = pv, then we get

$$\psi_{(x,t)} = Ae^{-\left(\frac{i}{h}\right)(Et-px)} \quad (2)$$

(Supriadi, 2022: 60).

The solution in the form of a solution to the wave function equation using the Schrodinger approach does not depend on time as in helium ions in spherical coordinates obtained in the following equation (3).

$$\frac{h^2}{2m}\frac{1}{r^2}[\frac{\partial}{\partial r}(r^2\ \frac{\partial}{\partial r}) + \frac{1}{sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{sin^2\theta}\ \frac{\partial}{\partial\phi^2}]\psi_{(r,\theta,\phi)} + (E-V)\psi_{(r,\theta,\phi)} = 0 \quad (3)$$

The solution of the above equation can use variable separation by introducing the function written as follows:

$$\psi_{(r,\theta,\phi)} = R(r)Y(\theta,\Phi) \quad (4)$$

Where $R(r) = e^{-\frac{r}{na0}}\left(\frac{2r}{na0}\right)^{l} L^{2l+1}_{n+1}(\frac{2r}{na0})$

(5)

$$Y(\theta,\Phi) = (-1)^{\frac{m+|m|}{2}}\sqrt{\frac{2l+1}{2\pi}\frac{(l-|m|)!}{(l+|m|)!}}e^{im\phi}P^{m}_{l}(cos\theta) \quad (6)$$

The above wave function is a wave function in position space that applies to all hydrogenic atoms. Then the above wave function can also be expressed in momentum space using the Fourier transform. The formulation for the Fourier transformation is stated as follows:

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int \varphi(p)\, e^{\frac{ipx}{\hbar}} dp \quad (7)$$

$$\varphi_{(p)} = \frac{1}{\sqrt{2\pi\hbar}} \int \psi(x) e^{-\frac{ipx}{\hbar}} dx \quad (8)$$

Equations (7) and (8) above are equations for the Fourier transform that applies to one-dimensional motion, while for motion in three dimensions using spherical coordinates, equations will be obtained:

$$\varphi_{n,l,m} = h^{-\left(\frac{3}{2}\right)} \int_0^\infty \int_0^\pi \int_0^{2\pi} e^{-\left(\frac{2\pi f}{h}\right)(\sin^2\theta \cos(\Phi-\phi)+\cos^2\theta)} \sqrt{\frac{1}{2\pi}}\, e^{\pm im\phi} P_l^m cos\theta \sqrt{\frac{2l+1}{2}\left(\frac{(l-|m|!)}{(l+|m|!)}\right)} \frac{(2\gamma)^{l+1}}{(n+l)!} \sqrt{\frac{\gamma(n-l-1)!}{(n+l)!}} e^{-(\gamma r)} r^l \left[L_{n+1}^{2l+1}(2\gamma r)\right] r^2 sin\theta\, dr d\theta d\phi$$

There are two types of wave functions owned by helium ions ($^4_2He^+$) including angular momentum functions ($Y(\Theta,\Phi)$) (Sakimoto, 2010) and radial momentum functions ($F_{n,l}(p)$) (Mehmood et al., 2021). So that it is obtained:

$$\varphi(p,\theta,\Phi) = \left\{\frac{1}{(2\pi)^{\frac{1}{2}}} e^{\pm im\phi} \sqrt{\frac{2l+1}{2}\left(\frac{(l-|m|!)}{(l+|m|!)}\right)} P_l^m cos\theta\right\} \left\{\frac{\pi(l)^2}{\left(\frac{z}{na_0}\right)^{\frac{3}{2}} (\hbar)^{\frac{3}{2}}} 2^{2l+4} l! \left(\frac{n(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}} \frac{\zeta^l}{(\zeta^2+1)^{l+2}} C_{n-l-1}^{l+1}\left(\frac{\zeta^2-1}{\zeta^2+1}\right)\right\} \quad (9)$$

(Podolsky et al., 1929).

So that the radial equation can be known:

$$F_{n,l}(p) = \frac{\pi(l)^2}{\left(\frac{z}{na_0}\right)^{\frac{3}{2}} (\hbar)^{\frac{3}{2}}} 2^{2l+4} l! \left(\frac{n(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}} \frac{\zeta^l}{(\zeta^2+1)^{l+2}} C_{n-l-1}^{l+1}\left(\frac{\zeta^2-1}{\zeta^2+1}\right)$$

(10) (Bethe, 1975: 39).

Methods (15%)

This research is research using the literature study method in physics theory, namely by developing pre-existing theories in physics theory related to hydrogenic atoms (having a single electron). There are several stages that need to be done in this research, namely the preparation stage, the theory development stage, the validation stage of the results, and the data collection stage.

The initial stage carried out in this research is the preparation stage by collecting from various literature, such as quantum physics books, core physics, modern physics, mathematical physics, and several journals related to helium ions and single-electron Schrodinger equations that are relevant to the research topics discussed.

Furthermore, the second stage is the stage of theory development from previously existing theories, both from literature and books related to the wave function in the developed position space so as to form a novelty. To express the wave function in position space on hydrogenic atoms, it can be expressed in the form of spherical coordinates as follows.

$$\left(\psi_{n,l,m}(r,\theta,\phi)\right) = \frac{(2\gamma)^{l+1}}{(n+l)!}\sqrt{\frac{\gamma(n-l-1)!}{n(n+l)!}}e^{-(\gamma r)}r^{l}\left[L_{n+1}^{2l+1}\right](2\gamma r)\left]\sqrt{\frac{2l+1}{2}\left(\frac{(l-|m|)!}{(l+|m|)!}\right)}\sqrt{\frac{1}{2\pi}}e^{\pm im\phi}p_{l}^{m}\cos\theta \quad (1)$$

The above equation is a form of function in position space that can be symbolized by (x, y, z/ n, l, m). To transform the function in position space to the function in momentum space, it can use the Fourier transform.

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}}\int \varphi(p)\, e^{\frac{ipx}{\hbar}} dp \quad (2)$$

$$\varphi(p) = \frac{1}{\sqrt{2\pi\hbar}}\int \varphi(x)\, e^{\frac{ipx}{\hbar}} dx \quad (3)$$

In reality, spherical coordinates are often used to describe the case of atoms and hydrogenic ions in three-dimensional space. In general, (x,y,z) is used to express the position space while (p_x, p_y, p_z) is used to query the momentum space. In spherical coordinates, it is known that the volume element is $eV = r^2 sin\theta dr d\theta d\phi$. If it is generalized, it can be written as follows.

$$x = r\sin\theta cos\phi \quad p_x = p\sin\theta cos\phi \quad (4)$$

$$y = r\sin\theta sin\phi \quad p_y = p\sin\theta sin\phi \quad (5)$$

$$z = r\,cos\phi \quad p_z = p\,cos\phi \quad (6)$$

With p is the size of the total momentum vector, while θ and ϕ is the orientation of the momentum vector relative to the cartesian coordinate axis. The result of the transformation of the position wave function to the transformation of the function is as follows.

$$\left(P_x, P_y, P_z,/n,l,m\right) = h^{-\left(\frac{3}{2}\right)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\left(\frac{2\pi f}{h}\right)(xp_x+yp_y+zp_z)}\,(x,y,z,/n,l,m)\,dxdydz \quad (8)$$

Then, if the function $\left(P_x, P_y, P_z,/n,l,m\right)$ is expressed in the function (p,θ,Φ) and denoted by:

$$\varphi_{n,l,m}(p,\theta,\Phi) = h^{-\left(\frac{3}{2}\right)}\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} e^{-\left(\frac{2\pi f}{h}\right)(sin\theta\; sin\theta\cos(\Phi-\phi)+cos\theta\; cos\theta)}\sqrt{\frac{1}{2\pi}}e^{\pm im\phi}P_l^m cos\theta$$
$$\sqrt{\frac{2l+1}{2}\left(\frac{(l-|m|!)}{(l+|m|!)}\right)}\frac{(2\gamma)^{l+1}}{(n+l)!}\sqrt{\frac{\gamma(n-l-1)!}{(n+l)!}}e^{-(\gamma r)}r^l[L_{n+1}^{2l+1}(2\gamma r)]r^2 sin\theta\; dr d\theta d\phi \quad (9)$$

Where $\gamma = \frac{4\pi^2\mu e^2 z}{nh^2} = \frac{z}{na_0}$, then from equation (9) will be obtained the hydrogenic atomic wave function in momentum space which can be written as follows.

$$\varphi(p,\theta,\Phi) = \left\{\frac{1}{(2\pi)^{\frac{1}{2}}}e^{\pm im\phi}\sqrt{\frac{2l+1}{2}\left(\frac{(l-|m|!)}{(l+|m|!)}\right)}P_l^m cos\theta\right\}\left\{\frac{\pi(l)^2}{\left(\frac{z}{na_0}\right)^{\frac{3}{2}}(\hbar)^{\frac{3}{2}}}2^{2l+4}l!\left(\frac{n(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}}\frac{\zeta^l}{(\zeta^2+1)^{l+2}}C_{n-l-1}^{l+1}\left(\frac{\zeta^2-1}{\zeta^2+1}\right)\right\} \quad (10)$$

Based on the above equation, it can be seen that there are two wave functions, namely the angular wave function and the radial wave function, where for the hydrogen atom the angular wave function that applies is:

$$Y_{lm}(\Theta,\Phi) = \left\{\frac{1}{(2\pi)^{\frac{1}{2}}}e^{\pm im\phi}\sqrt{\frac{2l+1}{2}\left(\frac{(l-|m|!)}{(l+|m|!)}\right)}P_l^m cos\theta\right\}$$

And the radial wave function equation:

$$F_{n,l}(p) = \left(\frac{\pi}{2}\frac{(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}}n^2 2^{2l+2}2!\left(\frac{n^l p^l}{(n^2p^2+1)^{l+2}}\right)C_{n-l-1}^{l+1}\left(\frac{n^2p^2-1}{n^2p^2+1}\right)$$

(Bethe, 1975:39).

The above function is a radial wave function that applies to Hydrogen atoms where the above function is a form of function that has been expressed in atomic units for momentum or has been divided by p0 (Bransden, 1983).
According to Podolsky and Pauling (1929) if known value $\zeta = \frac{2\pi p}{\gamma h} = \frac{np}{zpo}$ where po is the momentum of the electron in the Bohr orbit whose value is $p_{o = \frac{2\pi\mu e^2}{h}}.$ Based on this value, the radial momentum equation that has been expressed in atomic units for momentum will apply to any hydrogenic momentum. The wave function can be written as:

$$F_{nl} = \frac{2^{2l+4} l! \pi}{\left(\frac{z}{na_0}\right)^{\frac{3}{2}} (h)^{\frac{3}{2}}} \left(\frac{n(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}} \frac{(\frac{np}{zp_0})^l}{\left[(\frac{np}{zp_0})^2+1\right]^{l+2}} C_{n-l-1}^{l+1} \left(\frac{\left(\frac{np}{np_0}\right)^2-1}{\left(\frac{np}{np_0}\right)^2+1}\right) \quad (11)$$

Based on the above equation, if the known atomic number for Helium Ion (z) is 2, then the radial momentum wave for Helium Ion can be expressed as follows.

$$F_{nl} = \frac{2^{2l+\frac{5}{2}} . 3^{3l+\frac{5}{2}}}{\pi^{\frac{1}{2}}} n^2 l! \left(\frac{n(n-l-1)!}{(n+l)!}\right)^{\frac{1}{2}} n^l p^l \left(\frac{p_0^{\,l+\frac{5}{2}}}{\left[n^2p^2+9p_0^{\,2}\right]^{l+2}}\right) C_{n-l-1}^{l+1} \left(\frac{n^2p^2-9p_0^{\,2}}{n^2p^2+9p_0^{\,2}}\right) (12)$$

Furthermore, the third stage that can be done is a validation of the results obtained from the development of existing theories, then readjusted with literature books or previous studies. In the fourth stage, the data retrieval stage is carried out by determining the wave function of the Helium Ion with quantum number n ≤ 4, then numerical data retrieval can be carried out to obtain the graphical form of the radial function. After data collection is carried out, then discuss related to a more detailed discussion of the calculation of the wave function problem solving on helium ions in momentum space with quantum numbers n ≤ 4 through data analysis using methods, techniques, and theoretical foundations as a basis for obtaining research conclusions carried out to answer the formulation of existing problems.

Results And Discussion

Solving problems related to hydrogenic atoms can be solved through the Schrodinger equation. With this Schrodinger equation can be used to determine the characteristics of atoms using different quantum numbers, it can produce different wave functions. Where the wave function of the Helium Ion ($^4_2He^+$) in momentum space is the solution of the Helium Ion ($^4_2He^+$) equation obtained using the Schrodinger equation through the variable separation method in the form of a complex quantity function consisting of radial equations that only depend on the radius and angular equations that only depend on the angle. In other words, the acquisition of the Helium Ion wave function ($^4_2He^+$) at quantum number n ≤ 4 in momentum space using the time-independent Schrodinger equation is obtained from the Fourier transform of the wave function that is hydrogenic (has a single electron) in position space. Based on literature studies, as for the acquisition of data on the calculation of the value of the reduced mass for the Helium Ion ($^4_2He^+$) by using the equation $\mu = \frac{m_{He^+} x m_e}{m_{He^+} + m_e}$ and obtained the result of 9.108142883 x 10-31 kg. As for the calculation of the Bohr atomic radius of Helium Ion ($^4_2He^+$) obtained by using the previous reduced mass of 9.108142883 x 10-31 kg through the equation $r = \frac{n^2h^2}{Zm}\frac{4\pi\varepsilon_0}{e^2} = \frac{n^2h^2}{Z\mu\alpha}$ obtained a result of 0.528689400016386 Å. Then from the results of the development of hydrogenic atomic theory in the form of Helium Ion ($^4_2He^+$) in momentum space can use the Fourier transformation so that the results of the wave function simulation of Helium Ion ($^4_2He^+$) are obtained.

The electron wave function in helium ions can be determined through equation (12) by entering a combination of the main quantum number and orbital (n,l) values. For n = 1 and l = 0, we get : $\frac{2^5}{\sqrt{\pi}}\frac{p_0^{\frac{5}{2}}}{[p^2+4p_0^2]^2}$. For n = 2 and l = 1 we get : $\frac{2^{11}}{\sqrt{6\pi}}\,p_0^{\frac{7}{2}}\frac{p}{(4p^2+4p_0^2)^3}$. For n = 3 and l = 2, we get : $\frac{2^{11}3^4}{\sqrt{120\pi}}\,p^2\,.\,p_0^{\frac{9}{2}}\frac{1}{[9p^2+4p_0^2]^4}$. For n = 4 and l = 3, we get :

$\frac{2^{23}}{\sqrt{35\pi}}\,p^3\frac{Po^{\frac{11}{2}}}{(16p^2+4Po^2)^5}$. The complete wave function of helium ions can be presented in Table (1) below.

Table 1. *Radial Wave Function of Helium Ion ($^4_2He^+$) at n ≤4 in Momentum Space*

n	L	$F_{nl}(p)$
1	0	$\frac{2^5}{\sqrt{\pi}}\frac{p_0^{\frac{5}{2}}}{[p^2+4p_0^2]^2}$
2	0	$\frac{2^8}{\sqrt{2\pi}}\,p_0^{\frac{5}{2}}\frac{4p^2-4p_0^2}{(4p^2+4p_0^2)^3}$
	1	$\frac{2^{11}}{\sqrt{6\pi}}\,p_0^{\frac{7}{2}}\frac{p}{(4p^2+4p_0^2)^3}$
3	0	$\frac{864}{\sqrt{3\pi}}\,p_0^{\frac{5}{2}}\frac{3(81-120p_0^2p^2+16p_0^4)}{(9p^2-4p_0^2)^2}$
	1	$\frac{27648}{\sqrt{\pi}}\,p\,p_0^{\frac{7}{2}}\frac{9p^2-4p_0^2}{[9p^2+4p_0^2]^4}$
	2	$\frac{2^{11}3^4}{\sqrt{120\pi}}\,p^2\,.\,p_0^{\frac{9}{2}}\frac{1}{[9p^2+4p_0^2]^4}$
4	0	$\frac{2^{16}}{\sqrt{\pi}}\frac{Po^{\frac{5}{2}}(64p^2+108p^2po^2-po^2)}{(16p^2+4Po^2)^5}$
	1	$\frac{2^{14}p}{\sqrt{240\pi}}\frac{Po^{\frac{7}{2}}(160p^4-112p^2Po^2+10Po^2)}{(4p^2+Po^2)^5}$
	2	$\frac{2^{21}.3p^2}{\sqrt{720\pi}}\frac{Po^{\frac{9}{2}}(16p^2-4Po^2)}{(16\,p^2+4Po^2)^5}$
	3	$\frac{2^{23}}{\sqrt{35\pi}}\,p^3\frac{Po^{\frac{11}{2}}}{(16p^2+4Po^2)^5}$

Conclusion

Based on the theoretical study that has been done, it can be concluded that the wave function of the Helium Ion {({_2^4}He}^+) in momentum space n ≤ 4 using the Schrodinger equation in spherical coordinates which produces a complex wave function in the form of a radial equation obtained by transforming the wave function in position space can use the Fourier transform. The normalized radial wave function of the hydrogen atom at the main quantum number ≤ 4 produces 10 radial wave functions using the formula :

$$F_{nl} = \frac{2^{3l+5} l!(n)^2}{n^{\frac{1}{2}}} \left(\frac{(n-l-1)!^{\frac{1}{2}}}{(n+l)!} \right) n^l p^l x \left(\frac{P_0^{l+\frac{5}{2}}}{\left[n^2p^2+4p_0^2\right]^{l+2}} \right) C_{n-\,l-1}^{\,l+1} \left(\frac{n^2p^2-4p_0^2}{n^2p^2+4p_0^2} \right).$$

References

Alimah, Siti., Sriyono. 2016. Kajian Sistem Pemurnian Helium Reaktor HTGR Berdaya Kecil. *Jurnal Pengembangan Energi Nuklir* Vol 18 (2): 123-133.

Azis, S. A. dan Z. Abdullah. 2015. Teknik Pemisahan Operator dan Pendekatan Spektral sebagai Solusi Persamaan Schrodinger Bergantung Waktu pada Atom Hidrogen. Jurnal Fisika Unand. 4(3): 255-262.

1975, A. 2003. *Concept of Modern Physics.* Six Edition. United States of America: Tata Mcgraw-Hill Companies, Inc.

BH Bransden, CJ Joachain. 1983. Physics Of Atoms And Molecules Edisi Pertama. New York: John Wiley & Sons, Inc., New York.

Damayanti, D. D., Supriadi, B., & Nuraini, L. (2019). *No Title*. *4*(ISSN : 2527 – 5917), 252–257.

Fuadah, F., S.H.B. Prastowo., L. Nuraini. 2018. Solusi Persamaan Schrodinger Atom Deuterium dengan Bilangan Kuantum . Prosiding Seminar Nasional Pendidikan Sains 2018. 3(2): 142-147.

Krane, K. S. 2012. *Modern Physics Third Edition.*United States of America: John Wiley & Sons, Inc.

Hermanto, W. 2016. Fungsi Gelombang Atom Deuterium dengan Pendekatan Persamaan Schrodinger. Prosiding Seminar Nasional Pendidikan Sains 2016: 794-802.

Lutfin, N. A., & A, F. (2020). Metoda Hartree Fock Untuk Menentukan Energi Total Inti Helium-4. *PHYDAGOGIC Jurnal Fisika Dan Pembelajarannya*, *3*(1), 1–7. https://doi.org/10.31605/phy.v3i1.954

Makmum, M. S., Supriadi, B., & Prihandoko, T. (2020). Fungsi Gelombang Ion Helium Dalam Representasi. *Jurnal Pembelajaran Fisika*, *4*(4), 152–159.

Nurlina. (2017). Fisika Kuantum. *Gavamedia: Yogjakarta*, *October*, 1–214.

Podolsky, B., dan L. Pauling. 1929. The Momentum Distribution in Hydrogen- · Like Atoms. Physical Review 34: 109-116.

Purwanto, A. 2016. *Fisika Kuantum Edisi 2 Revisi*. Yogyakarta: Gava Media.

Sakimoto, K. 2010. Cross Sections for Antiproton Capture by Helium Ions. *Journal Physical Review Atomic, Molecular, and Optical Physics*. 82(1):1-12.

Saregar, Antomi. 2015. Analisis Spektrum Energi dan Fungsi Gelombang Potensial *Non-Central* menggunakan Supersimetri Mekanika Kuantum. *Jurnal Ilmiah Pendidikan Fisika*.Vol 4(2): 198-199.

Supriadi, B., Firdha, K. A. A., Nurul, M., dan Elsa, M. J. (2022). *Fisika Kuantum*. Jember: UPT Penerbit Universitas Jember.

Wijayanto, M. P. (2020). *Wave Function And Energy of Lithium Ion In The Presence of Electrostatic Field Based on Perturbation Theory*. January 2016.