**Jurnal Pendidikan Fisika dan Keilmuan (JPFK)**
Vol. 8, No 2. September 2022, pp. 212-225
P-ISSN: 2442-8868, E-ISSN: 2442-904X, DOI http://doi.org/10.25273/jpfk.v8i2.13626

# METHODS TO ACCELERATE EQUILIBRIUMIN OVERDAMPED BROWNIAN MOTION

**Nur Ainayah [*1], Iwan Setiawan [2], Dedy Hamdani [3]**

[123]Physics Education, Faculty of Teacher Training and Education, Bengkulu UniversityJalan WR Supratman, Bengkulu 38122, Indonesia

e-mail: [1]ainayah306@gmail.com, [2]iwansetiawan@unib.ac.id,[3] Dedy.hamdani@unib.ac.id

**Abstract**

Brownian motion is the random motion of nanoparticles in a fluid. If this motion is balanced, and then disturbed, it will take a very long time to return to *equilibrium.* Required a method in the form of additional energy to accelerate the process of evolution of these particles to reach the equilibrium point. To find this additional energy, we use a method of accelerating adiabatic quantum dynamics, known as *Shortcuts to Adiabaticity* (STA). This method will be applied using the Fokker-Planck equation and tested on overdamped Brownian motion. The research was conducted by reviewing various literatures that are in line with this research. By using this method, an additional term will be obtained in the form of an additional potential that depends on the friction constant and depends on time and is influenced by the initial potential. For the spring potential whose constant is replaced with lamda, we get additional additional potential results $U_1 = \frac{\gamma \dot{\lambda}}{4\lambda(t)} x^2$ and for shear potential with time dependent lamda,we get an additional potential of $u_1 = -\dot{\lambda}\gamma x$

**Key Words :** Brownian Motion; Fokker-Planck Equation; *Shortcuts to Adiabaticity (STA)*

***How to Cite***: Ainaya, N., Setiawan, I., Hamdani, D., (2022). Methods To Accelerate Equilibrium In Overdamped Brownian Motion. *Jurnal Pendidikan Fisika dan Keilmuan (JPFK)*, 8(2), 212-225. Doi: http://doi.org/10.25273/jpfk.v8i2.13626

## Introduction

Nowadays people always want every thing is done quickly so that the available time can be used more efficiently and effectively (Benggadinda & Setiawan, 2021). Likewise with the world of production (Masuda & Nakamura, 2010) (Setiawan, 2019) (Del Campo & Kim, 2019). They also want the shortest possible time (Vinjanampathy & Anders, 2016) (Takahashi, 2014). However, this shorter time should not change the form or content of the material in a system (Setiawan et al., 2017). A system in ordinary physics is viewed from two perspectives. First, a macroscopic view that can be seen from the nature of the system on a large scale using the senses, for example thermodynamics (Viridi & Khotimah, 2010). Second, a microscopic view consisting of some *probabilities* formed and influenced by matter with the same properties (Novita et al., nd). Examples are electrons, protons, neutrons, and atoms (Aini, 2020). The Brownian particles that we are going to consider also belong to systems with a microscopic view.

To accelerate a microscopic system, the properties and characteristics of the system need to be maintained (adiabatic state). In thermodynamic systems,

adiabatic is a process that does not require an exchange of heat/energy (H Kara, 2014). The adiabatic theorem in quantum systems is used for tracking the basic energy (*ground state)* (Santos & Sarandy, 2018) (Panati et al., 2002). For example, look for the time-dependent Hamiltonian, H(t) (Jing et al., 2016). Initially in the ground state, the controlled Hi then evolves adiabatically towards Hf or late Hamiltonian (Puebla et al., 2020)(Albash & Lidar, 2018). The transfer process whilemaintaining the properties and characteristics of the system can be carried out, but it takes a long time. This condition makes the evolution/displacement process not optimal, and there is even *noise* (disruption to the system). To overcome this situation, certain methods are needed. The methods currently being developed are *fast forward* and *shortcuts to adiabaticity (STA)*(Guéry-Odelin et al., 2019).

The *fast forward* was developed by Nakamura and Masuda on a relativistic system (Benggadinda & Setiawan, 2021) (Torrontegui et al., 2012) (Masuda & Nakamura, 2010) (Masuda & Nakamura, 2011) and has been applied to a Carnot engine combined with an oscillator time dependent harmonics (Nakamura et al., 2020). Meanwhile, the *shortcuts to adiabaticity* (STA) method was developed by scientists Gonzale Muga, Xi Chen, and Del Campo. STA is used to accelerate adiabatic quantum dynamics through quantum phase transitions (Del Campo, 2013)(Saberi et al., 2014)(Hatomura, 2017). STA is applied by providing a temporary Hamiltonian (Couvert et al., 2008)(Masuda & Nakamura, 2010). STA is atime dependent protocol in the presence of control parameters to produce the same end result in a short time (Chen et al., 2010) (Saberi et al., 2014) (Torrontegui et al., 2013)(Torrontegui et al., 2017). The STA method can also be applied to reverseengineer a quantum measurement process enhancement that depends on control flexibility (Cárdenas-López & Chen, 2022)(Chen et al., 2010).

In this study, the STA method was applied to particles that move randomly (Brown motion). These particles are considered in an isothermal state. If this particle is given an external potential, it will take a long time to return to *equilibrium* (Li et al., 2017). The purpose of this study was to obtain a method in the form of additional energy so that the time to return to a state of equilibrium would be shorter (Patra & Jarzynski, 2017). The equation that will be used today is the Fokker-Planck equation, which is a partial differential equation that describes the distribution function of particles in a system containing many colliding nanoparticles(Malik & Malang, 2006).

## Methods

The current research is a quantitative theory of physics development research. The method used in this research is a literature study. The literature study was carried out related to the *Shortcuts to Adiabaticity* (STA) method as a determinant of control parameters, Brownian motion, and large damping or *Overdamped forms* and the development of the Fokker-Planck Equation to calculate additional potential so that Brownian particles can return to their equilibrium state quickly. This research was conducted from May 2022 to July 2022at Bengkulu University.

This study is based on previous experimental studies related to the ESE protocol on Brownian motion. Initially the particles are in equilibrium, then the

temperature is increased so that the particles move randomly again. If this is allowed, then the particles will take a very long time to return to equilibrium. That's why scientists developed the ESE protocol and it has been proven to be able to make equilibrium particles 100 times faster than before. In the experiments that have been carried out, namely changing the volume of gas by using a piston which takes a very long time to re-equilibrate. Thus, a temperature-related control protocol known as ESE was developed. In order to really shorten the time, the energy in the system is also analyzed in depth. The steps taken are increasing confinement to increase positive work. This is in stark contrast to the adiabatic transformation where the calorific value increases monotonically, because the system dissipates heat throughout the protocol. (Martínez et al., 2016).

The next experiment was carried out using silica microspheres with a radius of 1μm which was diluted with water and then transferred to a three-dimensional space. Then, an infrared laser beam with a maximum power of 500 mW is extended and injected through an oil-immersed object (Leica, 63×NA 1.40). The laser power is trapped and modulated by an external voltage. This is able to make the Brownian particles reach equilibrium more quickly. The acceleration that occursalso depends on the friction provided by the fluid in question (Martínez et al., 2016).Based on this, this study tries to find additional potential terms using the *shortcuts to adiabaticity* (STA) method with an initial potential that is different from previous studies. The results obtained are also in line with previous research, which is ableto accelerate Brownian motion to reach equilibrium.

In order to avoid different interpretations in this journal, an operational definition of each variable is needed. The variables to be studied include,

a. Method *Shortcuts to Adiabaticity* (STA)

   STA was initially used to achieve final results in a short time in adiabatic conditions. However, it has now been extended to search for adiabatic shortcuts (Plata et al., 2021) (Guéry-Odelin et al., 2019) and developed in quantum dynamics. The STA method aims to develop a protocol so that the particles quickly return *equilibrium* with a record of events occurring in adiabatic conditions (Chen et al., 2010) (Papoular & Stringari, 2015) and isolated systems (Schaff et al., 2011). To speed up the*equilibration* and increase power, a methodknown as *Engineered Swift Equilibration* (ESE) (Martínez et al., 2016).

b. Brownian

   motion Brownian motion is the random motion of microscopic particles dissolved in a fluid. Based on Robert Brown's research, two results about Brownian motion were obtained, namely the irregularity of the trajectory of the particles in the fluid and the absence of the effect of the interaction of two different particles (Romadani & Rosyid, 2022). Then Gouy added a few things. That is, the faster the motion the smaller the particles, the motion is affected by the density and composition of the particles, the fluid with a small viscosity accelerates the Brownian motion, high temperatures accelerate the motion, and the particle motion never stops (Najmudin, 2018). Brownian motion can be observed in nanoparticles that are in a certain fluid which is inhibited by fluid viscosity and stochastic forces. This is caused by the collision of molecules that move randomly in the fluid (Pal & Deffner, 2020).

c. Stochastic Dynamics Stochastic

processes are included in the physical sciences related to dynamics. Because, physics can be expressed as an effort to realize mathematical concepts into contextual or scientific concepts that contain numbers (Physics Education, 2019). The stochastic dynamics system uses a lot of modeling data along with a system for future masses (Apriliani, 2004). Stochastic dynamics systems or also known as stochastic differentials can be applied to systems that move randomly, especially non-relativistic systems (Jannah, 2016).

*d. Overdamped*

Perfect damping/overdamped is a condition where the particle has stopped moving before reaching its equilibrium point. This can happen due to excessive damping in the system. This damping can be in the form of air friction, other friction that opposes the direction of motion of objects, or comes from an known as overdamping. This happens because the system continues to add damping until it passes its critical point (Rusianto & Susastriawan, 2021)

e. Fokker-Planck equation

As the name implies, the Fokker-Planck equation was first introduced by Fokker and Planck. This equation is usually applied to particle balance systems (Frank, 2005). The Fokker-Planck equation is defined as an implementation of a partial differential equation to describe the random motion of particles in a fluid (Brown motion). The Fokker-Planck equation in stochastic dynamics describes the movement of the probabilistic density function (Romadani & Rosyid, 2022) and the shift in velocity space caused by the collision of the evolving particles (Palupi, 2010).

To review the Brownian motion in the *overdamped* the Fokker-Planckequation is used as follows!

$$\frac{\partial \rho}{\partial t} = \frac{1}{\gamma}\frac{\partial}{\partial x}\left[\frac{\partial U}{\partial x}\rho + \frac{1}{\beta}\frac{\partial \rho}{\partial x}\right] \quad (1)$$

Where *U* is a variable that represents the potential of the system, $\rho$ is avariable that describes the distribution of particles, $\gamma$ is a friction constant, $\beta$ is a thermodynamic constant that relates to temperature, whose magnitude is $\beta = -\frac{1}{KT}$ *x* is a variable that represents particle position/particle location, and t is time.

If we consider the distribution of particles at *equilibrium* or $\rho(x,\lambda)$ independent of time $\left(\frac{\partial \rho}{\partial t} = 0\right)$, as well as their initial potential or $U_0 = \frac{1}{2}\lambda x^2$, with λλ as a control paramenter, then we get :

$$0 = \frac{1}{\gamma}\frac{\partial}{\partial x}\left[\lambda x\, \rho + \frac{1}{\beta}\frac{\partial \rho}{\partial x}\right] \quad (2)$$

and

$$-\frac{1}{\beta}\frac{\partial^2 \rho}{\partial x^2} = \frac{\partial \lambda x\, \rho}{\partial x} \quad (3)$$

If the two terms in equation (3) are integrated , then we get the particle distribution $\rho(x,\lambda)$ as follows,

$$\rho(x,\lambda) = C\, exp\left(-\frac{\beta\,\lambda x^2}{2}\right) \quad (4)$$

The velue of the constant C can be obtained by means of the following normalization!

$$\int_{-\infty}^{\infty} |\rho(x,\lambda)|^2\, dx = 1 \quad (5)$$

By substituting the result of $\rho(x,\lambda)$ equation (4) into equation (5), the value of constant C is obtained, namely:

$$C = \sqrt{\frac{\beta\,\lambda}{\pi}} \quad (6)$$

Then, substituting the value of constant C above, then the distribution of particles ina state of *equilibrium* can be written as ,

$$\rho(x,\lambda) = \sqrt{\frac{\beta\,\lambda}{\pi}}\, exp\left(-\frac{\beta\,\lambda x^2}{2}\right) \quad (7)$$

The particle distribution pattern ($\rho(x,\lambda)$) for x ranging from *x= -2* to *x=2* can be observed in the graph below!

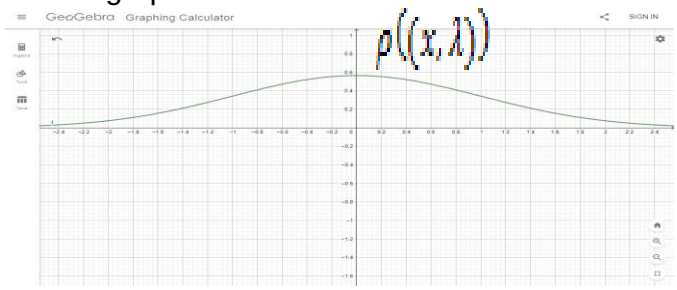


**Figure 2. Particle distribution starting from x= -2 to x = 2, when and = 1**

Furthermore, if the system is disturbed with the control parameters $\lambda\,\lambda$ being considered as time-dependent control parameters ($\lambda(t)$), then equation (7) can be written as:

$$\rho(x,\lambda(t)) = \sqrt{\frac{\beta\,\lambda(t)}{\pi}}\, exp\left(-\frac{\beta\,\lambda(t)x^2}{2}\right) \quad (8)$$

So $\rho(x,\lambda(t))$ that equation (8) still fulfills the Fokker-Planck equation in equation (1), then the potential of the system is written as $U = U_0 + U_1$,so that:

$$U = \frac{1}{2}\lambda(t)x^2 + U_1 \quad (9)$$

With $U_1$ is the amount of additional potential that must be added to the system so that the distribution of particles to return to an *equilibrium* can be faster. This additional potential is the main problem that must be resolved in this study.

By substituting $\rho(x,\lambda(t))$ in equation (8) and U in equation (9) into the Fokker-Planck equation in equation (1), the Fokker-Planck equation in equation (1) can be written as:

$$\frac{\partial\rho(x,\lambda(t))}{\partial t} = \frac{1}{\gamma}\frac{\partial}{\partial x}\left[\left(\lambda(t)x + \frac{\partial U_1}{\partial x}\right)\rho(x,\lambda(t)) + \frac{1}{\beta}\frac{\partial\rho(x,\lambda(t))}{\partial x}\right] \quad (10)$$

If on the left side $\rho(x,\lambda(t))$ derived with respect to t, then the chain derivative is obtained as follows:

$$\frac{\partial\rho(x,\lambda(t))}{\partial t} = \frac{\partial\rho(x,\lambda(t))}{\partial\lambda}.\frac{\partial\lambda}{\partial t} = \frac{\partial\rho(x,\lambda(t))}{\partial\lambda}.\dot{\lambda} \quad (11)$$

Solve equation (11) until the following results are obtained,

$$\frac{\partial\rho(x,\lambda(t))}{\partial t} = \left[\frac{1}{2\lambda(t)} - \frac{\beta}{2}x^2\right]\rho(x,\lambda(t)).\dot{\lambda} \quad (12)$$

Then, solve the equation on the right side of equation (10). Thus, the results areobtained, namely:

$$\begin{aligned}\frac{1}{\gamma}\left[\frac{\partial}{\partial x}\lambda(t)x\rho(x,\lambda(t)) + \frac{\partial}{\partial x}\frac{\partial U_1}{\partial x}\rho(x,\lambda(t)) + \frac{1}{\beta}\frac{\partial^2\rho(x,\lambda(t))}{\partial x^2}\right] \quad (13)\\ = \frac{1}{\gamma}\left[[\lambda(t) - \lambda(t)^2\beta x^2]\rho(x,\lambda(t)) + \left[\frac{\partial^2 U_1}{\partial x^2} - \frac{\partial U_1}{\partial x}\beta\lambda(t)x\right]\rho(x,\lambda(t))\right.\\ \left. + \frac{1}{\beta}\beta^2\lambda(t)^2x^2\rho(x,\lambda(t))\right]\end{aligned}$$

After that, substitute the results obtained in equation (12) and (13) into equation(10). So, we get:

$$\begin{aligned}\left[\frac{1}{2\lambda(t)} - \frac{\beta}{2}x^2\right]\rho(x,\lambda(t)).\dot{\lambda}\\ = \frac{1}{\gamma}\left[[\lambda(t) - \lambda(t)^2\beta x^2]\rho(x,\lambda(t)) + \left[\frac{\partial^2 U_1}{\partial x^2} - \frac{\partial U_1}{\partial x}\beta\lambda(t)x\right]\rho(x,\lambda(t))\right.\\ \left. + \frac{1}{\beta}\beta^2\lambda(t)^2x^2\rho(x,\lambda(t))\right] \quad (14)\end{aligned}$$

Then, solve the above equation. Thus, we get the equation :

$$\left[\frac{1}{2\lambda(t)} - \frac{\beta}{2}x^2\right].\gamma\dot{\lambda} = \left[\frac{\partial^2 U_1}{\partial x^2} - \frac{\partial U_1}{\partial x}\beta\lambda(t)x\right] + \lambda(t) \quad (15)$$

If the above equation is rewritten into twi equations, the we get :

$$\lambda(t) = 0$$

(16)

and

$$\left[\frac{1}{2\lambda(t)}-\frac{\beta}{2}x^2\right].\gamma\dot{\lambda} = \left[\frac{\partial^2 U_1}{\partial x^2}-\frac{\partial U_1}{\partial x}\ \beta\,\lambda(t)\,x\right] \quad (17)$$

Equation (17) can be rewritten into two equations, namely:

$$\frac{\gamma\,\dot{\lambda}}{2\lambda(t)} = \frac{\partial^2 U_1}{\partial x^2} \quad (18)$$

and

$$-\frac{\beta\gamma\,\dot{\lambda}}{2}x^2 = -\frac{\partial U_1}{\partial x}\ \beta\,\lambda(t)x \quad (19)$$

Both equations Each of the above will get a solution for an additional potential or $U_1$ equal to,

$$U_1 = \frac{\gamma\,\dot{\lambda}}{4\lambda(t)}\ x^2 \quad (20)$$

If we consider the initial potential to be $U_0 = U(x)$ and consider the distribution of particles in an *equilibrium* or $\rho(x,\lambda)$ independent of time, $\left(\frac{\partial\rho}{\partial t}=0\right)$, then the Fokker – Planck equation in equation (1) becomes:

$$\frac{\partial\rho}{\partial t} = \frac{1}{\gamma}\frac{\partial}{\partial x}\left[\frac{\partial U(x)}{\partial x}\rho+\frac{1}{\beta}\frac{\partial\rho}{\partial x}\right] \quad (21)$$

and

$$0 = \frac{1}{\gamma}\frac{\partial}{\partial x}\left[\frac{\partial\,U(x)}{\partial x}\rho+\frac{1}{\beta}\frac{\partial\rho}{\partial x}\right] \quad (22)$$

After that, solve equation (22) to obtain the particle distribution equation $\rho(x,\lambda)$, namely:

$$\rho(x,\lambda) = C\ exp\left(-\beta\ U(x)\right) \quad (23)$$

To get the value of the constant C, it can be searched by normalization. In this case, normalization is only limited to positive U(x). So the system doesn't explode. Thus, the normalization limit starts from 0 to $\infty$ as in equation (24) below:

$$\int_0^\infty |\rho(x,\lambda)|^2\,dx = 1 \quad (24)$$

By plugging $\rho(x,\lambda)$ in equation (23) into equation (24) and then completing the integral process, the constant C value is obtained by:

$$C = \sqrt{2\beta} \quad (25)$$

After that, substitute The value of C that has been obtained is converted to equation (23), so that the particle distribution equation can be written as:

$$\rho(x,\lambda) = \sqrt{2\beta}\ exp\left(-\beta U(x)\right) \quad (26)$$

The particle distribution pattern ($\rho(x,\lambda)$) can be observed in the graph below!

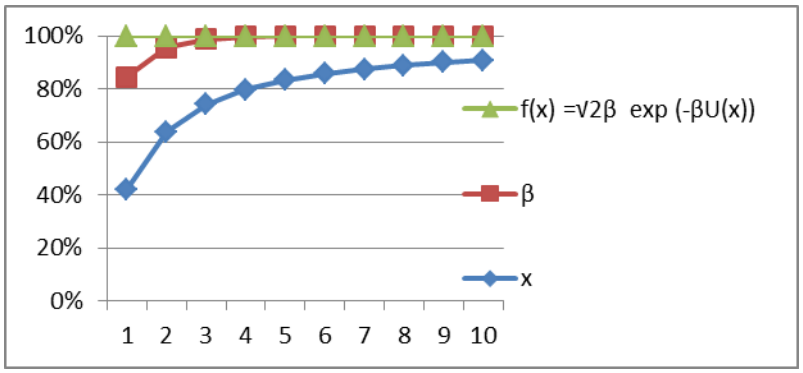


**Figure 3. Particle distribution from x = 1 to x= 10, when = 1**

Furthermore, if the system is disturbed by changing the initial $U_0 = U(x)$ potential into a time-dependent potential, $U_0 = u(x - \lambda(t))$, then the particle distribution in equation (26) can be written as:

$$\rho(x,\lambda(t)) = \sqrt{2\beta}\, exp\,(-\beta u(x - \lambda(t))) \qquad (27)$$

In order $\rho(x,\lambda(t))$ in equation (27) still fulfills the Fokker-Planck equation in equation (1), then the system potential can be written as $U = U_0 + u_1$. With $u_1$ is an additional potential that is the main goal of this study. This additional potential is useful for accelerating Brownian motion to reach a state of *equilibrium* again. With this additional potential, the potential U can be written:

$$U = u(x - \lambda(t)) + u_1 \qquad (28)$$

Then, the value of U that has been added to the potential in equation (28) and the value $\rho(x,\lambda(t))$ in equation (27) is substituted into the Fokker-Planck equation in equation (1). Thus, the following equation is obtained:

$$\frac{\partial \rho(x,\lambda(t))}{\partial t} = \frac{1}{\gamma}\frac{\partial}{\partial x}\left[\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_1}{\partial x}\right)\rho(x,\lambda(t)) + \frac{1}{\beta}\frac{\partial \rho(x,\lambda(t))}{\partial x}\right] \qquad (29)$$

If the left-hand side is solved by decreasing with $\rho(x,\lambda(t))$ respect to time, then the following chain derivative is obtained,

$$\frac{\partial \rho(x,\lambda(t))}{\partial t} = \frac{\partial \rho(x,\lambda(t))}{\partial u}.\frac{\partial u(x-\lambda(t))}{\partial \lambda}.\frac{\partial \lambda}{\partial t} = \frac{\partial \rho(x,\lambda(t))}{\partial u}.\frac{\partial u(x-\lambda(t))}{\partial \lambda}.\dot{\lambda} \qquad (30)$$

Then, solve the chain derivative in equation (30), until the result is:

$$\frac{\partial \rho(x,\lambda(t))}{\partial t} = \beta\rho(x,\lambda(t))\,\dot{\lambda} \qquad (31)$$

After that, complete the derivative on the right hand side. So, we get the result :

$$\frac{1}{\gamma}\left[\frac{\partial}{\partial x}\frac{\partial u_x}{\partial x}\rho(x,\lambda(t))+\frac{\partial}{\partial x}\frac{\partial u_1}{\partial x}\rho(x,\lambda(t))+\frac{1}{\beta}\frac{\partial^2\rho(x,\lambda(t))}{\partial x^2}\right] = \frac{1}{\gamma}\left[\left[\frac{\partial^2 u_x}{\partial x^2}-\frac{\partial u_x}{\partial x}\ \beta\right]\rho(x,\lambda(t))+\left[\frac{\partial^2 u_1}{\partial x^2}-\frac{\partial u_1}{\partial x}\ \beta\right]\rho(x,\lambda(t))+\frac{1}{\beta}\beta^{\,2}\rho(x,\lambda(t))\right] \quad (32)$$

Then, substituting equation (31) and equation (32) into equation (29), then we getthe following equation,

$$\beta\rho(x,\lambda(t))\,\dot{\lambda} = \frac{1}{\gamma}\left[\left[\frac{\partial^2 u_x}{\partial x^2}-\frac{\partial u_x}{\partial x}\ \beta\right]\rho(x,\lambda(t))+\left[\frac{\partial^2 u_1}{\partial x^2}-\frac{\partial u_1}{\partial x}\ \beta\right]\rho(x,\lambda(t))+\frac{1}{\beta}\beta^{\,2}\rho(x,\lambda(t))\right] \quad (33)$$

Solve the above equation until we get the following equation,

$$\dot{\lambda}\gamma = \left[-\frac{\partial u_x}{\partial x}-\frac{\partial u_1}{\partial x}+1\right]\beta+\left[\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_1}{\partial x^2}\right] \quad (34)$$

Next,above equation can be rewritten into two equations, namely:

$$\left[\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_1}{\partial x^2}\right]=0 \quad (35)$$

and

$$\beta\,\dot{\lambda}\gamma = \left[-\frac{\partial u_x}{\partial x}-\frac{\partial u_1}{\partial x}+1\right]\beta \quad (36)$$

then, solve equation (36) until the following equation is obtained,

$$-1+\dot{\lambda}\gamma = -\frac{\partial u_x}{\partial x}-\frac{\partial u_1}{\partial x} \quad (37)$$

We can rewrite the above equation into two equations, namely:

$$-1 = -\frac{\partial u_x}{\partial x} \quad (38)$$

And

$$\dot{\lambda}\gamma = -\frac{\partial U_1}{\partial x} \quad (39)$$

Since the main objective of this study is to find additional potential values or $U_1$ , then simply solve equation (39), because equation (39) contains a variablle. Therefore, solve equation (39) until the following result is obtained:

$$U_1 = -\dot{\lambda}\gamma x \quad (40)$$

Thus, to accelerate the system back to equilibrium, an additional potential which depends on the fluid friction constant ($\gamma$) and depends on time is required.

## CONCLUSION

Research has been carried out using the *Shortcuts to Adiabaticity* (STA) method which is applied in conjunction with the Fokker-Planck equation on*overdamped*which aims to obtain a method in the form of additional energy to accelerate the excessively damped Brownian motion to reach the equilibrium point. In this study, additional terms have been obtained from the excessively damped Brownian motion to accelerate the motion to reach the equilibrium point again. From the beginning it continued to move randomly without stopping and it took a long time to get back into balance. By adding additional energy in the form of this additional potential, it can accelerate Brownian motion towards equilibrium again in an adiabatic state without changing the properties and characteristics of the particle. Based on the ESE protocol the speed can be 100 times faster than before. From the results of this study obtained two results. First, if the initial potential of the system is $\frac{1}{2}\lambda(t)x^2$, an additional potential is obtained $U_1 = \frac{\gamma\lambda}{4\lambda(t)}x^2$. secondly, if the initial potential of the system is u(x-λ(t)), we get an additional potential of $u_1 = -\dot{\lambda}\gamma x$

## REFERENCES

Aini, NR (2020). *History of the Development of Physics (Quantum) from Classicalto Modern*. *4*(3), 57–71. https://doi.org/10.13140/RG.2.2.17085.08160

Albash, T., & Lidar, DA (2018). Adiabatic quantum computation. *Reviews of Modern Physics*, *90*(1). https://doi.org/10.1103/RevModPhys. 90.015002

Apriliani, E. (2004) Methods of Data Assimilation as Estimation of Solving Environmental Problems *Limits: Journal of Mathematics and Its Applications*, *1*(1), 14. https://doi.org/10.12962/j1829605x.v1i1 . 1345

Benggadinda, A., & Setiawan, I. (2021). Fast Forward Method to Accelerate Adiabatic Quantum Dynamics on Single Spin. *JST (Journal of Science and Technology)*,*10*(2),274–280. https://doi.org/10.23887/jstundiksha.v10i2.39876

Cárdenas-López, FA, & Chen, X. (2022). *Shortcuts to Adiabaticity for Fast Qubit Readout in Circuit Quantum*3. *Electrodynamics*http://arxiv.org/abs/2201.06007

Chen , X., R uschhaupt, A., Schmidt, S., Del Campo, A., Guéry-Odelin, D., & Muga, JG (2010). Fast optimal frictionless atom cooling in harmonic traps: Shortcut to adiabaticity. *Physical Review Letters*, *104*(6), 1–4. https://doi.org/10.1103/PhysRevLett.104.063002

Couvert, A., Kawalec, T., Reinaudi, G., & Guéry-Odelin, D. (2008). Optimal transport of ultracold atoms in the non-adiabatic regime. *Epl*, *83*(1). https://doi.org/10.1209/0295-5075/83/13001

Del Campo, A. (2013). Shortcuts to adiabaticity by counterdiabatic driving. *PhysicalReview Letters*, *111*(10), 1–5. https://doi.org/10.1103/PhysRevLett.111.100502

Del Campo, A., & Kim, K. (2019). Focus on Shortcuts to Adiabaticity. *New Journal of Physics*, *21*(5), 1–8. https://doi.org/10.1088/1367-2630/ab1437

Frank, TD (2005). Delay Fokker-Planck equations, perturbation theory, and data analysis for nonlinear stochastic systems with time delays. *Physical Review E- Statistical, Nonlinear, and Soft Matter Physics*, *71*(3), 1–14.https://doi.org/10.1103/PhysRevE.71.031106

Guéry-Odelin, D., Ruschhaupt, A., Kiely, A., Torrontegui, E., Martínez-Garaot, S., & Muga, JG (2019). Shortcuts to adiabaticity: Concepts, methods, and applications. *Reviews of Modern Physics*,*91*(4). https://doi.org/10.1103/RevModPhys.91.045001

H Kara, OAMA (2014). Application of the Principles and Laws of Physics on the Air-Blast Freezer Machine in Sardine Production at CV Indo Jaya Pratama. *Paper Knowledge . Toward a Media History of Documents*, *7*(2), 107–115.

Hatomura, T. (2017). Shortcuts to adiabaticity in the infinite-range ising model by mean-field counter-diabatic driving. *Journal of the Physical Society of Japan*, *86*(9), 1–7. https://doi.org/10.7566/JPSJ.86.094002

Jannah, W. (2016). THE FORMULATION OF STOCHASTIC DIFFERENTIAL.*Journal of Physics A: Mathematical and Theoretical*.

Jing, J., Sarandy, MS, Lidar, DA, Luo, DW, & Wu, LA (2016). Eigenstate tracking in open quantum systems. *Physical Review A*, *94*(4), 1–13. https://doi.org/10.1103/PhysRevA.94.042131

Li, G., Quan, HT, & Tu, ZC (2017). Shortcuts to isothermality and nonequilibrium work relations. *Physical Review E*, *96*(1), 1–12. https://doi.org/10.1103/PhysRevE.96.012144

Malik, M., & Malang, I. (2006). *SOLUTION OF FOKKER-PLANCK PARTIAL DIFFERENTIAL EQUATIONS WITH Siti Muyassaroh's LINE METHOD*.

Martínez, IA, Petrosyan, A., Guéry-Odelin, D., Trizac, E., & Ciliberto, S. (2016). Engineered swift equilibration of a Brownian particle. *Nature Physics*, *12*(9), 843–846. https://doi.org/10.1038/nphys3758

Masuda, S., & Nakamura, K. (2010). Fast-forward of adiabatic dynamics in quantum mechanics. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *466*(2116), 1135–1154. https://doi.org/10.1098/rspa.2009.0446

Masuda, S., & Nakamura, K. (2011). Acceleration of adiabatic quantum dynamics in electromagnetic fields. *Phys. Rev. A*, *84*(4), 43434. https://doi.org/10.1103/PhysRevA.84.043434 Najmudin

, I. (2018). *The study of relativistic brown motion processes using the Hanggi-Klimontovich approach*.

Nakamura, K., Matrasulov, J., & Izumida, Y. (2020). Fast-forward approach to stochastic heat engine. *Physical Review E*, *102*(1), 1–14.https://doi.org/10.1103/PhysRevE.102.012129

Novita, JS, Dewi, SN, Mufdhila, AR, Amalia, A., Viridi, S., Keilmuan, K., Nuclear, F., Mathematics, F., Alam, P., & Bandung, IT (nd). *Macro and Micro Circumstances with Statistical Physics as Initial Configuration for Optimizing the Congregational Prayer Shaf using Agent Based Modeling*. 187–194.

Pal, PS, & Deffner, S. (2020). Stochastic thermodynamics of relativistic Brownian motion. *New Journal of Physics*, *22*(7). https://doi.org/10.1088/1367-2630/ab9ce6

Palupi, DS (2010). The Fokker Planck Equation and Its Application in Astrophysics.*Physics Periodic*, *13*(2), A1-A6–A6. https://ejournal.undip.ac.id/index.php/berkala_fisika/article/view/2995/2679

Panati, G., Spohn, H., & Teufel, S. (2002). Space-Adiabatic Perturbation Theory in Quantum Dynamics. *Physical Review Letters*, *88*(25), 4. https://doi.org/10.1103/PhysRevLett.88.250405

Papoular, DJ, & Stringari, S. (2015). Shortcut to Adiabaticity for an Anisotropic Gas Containing Quantum Defects. *Physical Review Letters*, *115*(2), 1–5. https://doi.org/10.1103/PhysRevLett.115.025302

Patra, A., & Jarzynski, C. (2017). Shortcuts to adiabaticity using flow fields. *New Journal of Physics*, *19*(12). https://doi.org/10.1088/1367-2630/aa924c

Physics Education. (2019). *The 2019 Physics Education National Seminar on ThinkPair Share (Tps) Cooperative Model with the 2019 Physics Education NationalSeminar Approach*. *4*(1), 152–158.

Plata, CA, Prados, A., Trizac, E., & Guéry-Odelin, D. (2021). Taming the Time Evolution in Overdamped Systems: Shortcuts Elaborated from Fast-Forward and Time-Reversed Protocols. *Physical Review Letters*, *127*(19), 1–6. https://doi.org/10.1103/PhysRevLett.127.190605

Pradana, IA (2018). *RANDOM DECREMENT METHOD TO REDUCE THE MOVEMENT OF OBJECTS* (2018th ed.). Ten November Institute ofTechnology.

Puebla, R., Deffner, S., & Campbell, S. (2020). Kibble-Zurek scaling in quantum speed limits for shortcuts to adiabaticity. *Physical Review Research*, *2*(3), 32020. https://doi.org/10.1103/PhysRevResearch.2.032020

Romadani, A., & Rosyid, MF (2022). The process of relativistic diffusion through the Langevin and Fokker-Planck equations. *Journal of Teknosains*, *11*(2), 101. https://doi.org/10.22146/teknosains.63229Russianto

, T., & Susastriawan, AAP (2021). *Mechanical Vibration*.

Saberi, H., Opatrný, T., Mølmer, K., & Del Campo, A. (2014). Adiabatic tracking of quantum many-body dynamics. *Physical Review A - Atomic, Molecular, andOptical Physics*, *90*(6), 1–5. https://doi.org/10.1103/PhysRevA.90.060301

Santos, AC, & Sarandy, MS (2018). Generalized shortcuts to adiabaticity and enhanced robustness against decoherence. *Journal of Physics A: Mathematical and Theoretical*, *51*(2). https://doi.org/10.1088/1751-8121/aa96f1

Schaff, JF, Song, XL, Capuzzi, P., Vignolo, P., & Labeyrie, G. (2011). Shortcut to adiabaticity for an interacting Bose-Einstein condensate. *Epl*, *93*(2), 1–5. https://doi.org/10.1209/0295-5075/93/23001

Setiawan, I. (2019). Accelerated Adiabatic Quantum Spin Dynamics in Landau-Zener Model and Ising Model. *Journal of Coil Physics*, *2*(1), 57–64. https://doi.org/10.33369/jkf.2.1.57-64

Setiawan, I., Eka Gunara, B., Masuda, S., & Nakamura, K. (2017). Fast forward of the adiabatic spin dynamics of entangled states. *Physical Review A*, *96*(5), 1–13. https://doi.org/10.1103/PhysRevA.96.052106

Takahashi, K. (2014). Fast-forward scaling in a finite-dimensional Hilbert space. *Physical Review A*, *89*. https://doi.org/10.1103/PhysRevA.89.042113

Torrontegui, E., Ibáñez, S., Martínez-Garaot, S., Modugno, M., del Campo, A., Guéry-Odelin, D., Ruschhaupt, A., Chen, X., & Muga, JG (2013). Shortcuts to Adiabaticity. *Advances in Atomic, Molecular and Optical Physics*, *62*(September), 117–169. https://doi.org/10.1016/B978-0-12-408090-4.00002-5

Torrontegui, E., Lizuain, I., González-Resines, S., Tobalina, A., Ruschhaupt, A., Kosloff, R., & Muga, JG (2017). Energy consumption for shortcuts to adiabaticity. *Physical Review A*, *96*(2), 1–8.https://doi.org/10.1103/PhysRevA.96.022133

Torrontegui, E., Martínez-Garaot, S., Ruschhaupt, A., & Muga, JG (2012). *Shortcuts to adiabaticity: fast-forward approach*. *1*, 1–7. https://doi.org/10.1103/PhysRevA.86.013601

Vinjanampathy, S., & Anders, J. (2016). Quantum thermodynamics. *Contemporary Physics*, *57*(4), 545–579. https://doi.org/10.1080/00107514.2016.1201896

Viridi, S., & Khotimah, SN (2010). Statistical Physics Lecture Notes. *Physics Study Program, FMIPA ITB*, *August*. file:///D:/Research Methodology(Bayu Sugara)/1317-2857-1-PB.pdf