Didactic Anticipation of Two Matrix Multiplication Learning

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Abstract: This study aims to create a didactic design in an effort to overcome the learning obstacle through DDR (Didactical Design Research) in learning multiplication of two matrices through prospective, metapedadidactic, and retrospective analysis stages. Data collection using the following techniques: 1) observation, namely observing learning activities, analyzing teaching materials and learning implementation plans used by teachers in classroom learning; 2) tests in the form of diagnostic tests for students to determine the learning obstacle, prerequisite ability tests, and final identification tests; 3) interviews with mathematics subject teachers by asking questions that lead to student difficulties and teacher difficulties in delivering mathematics learning material; and 4) instructional videos in the learning process. The data were analyzed to find a learning obstacle and based on these findings, awas prepared Hypothetical Learning Trajectory (HLT) which was outlined in a hypothetical didactic design. The didactic design that has been designed is implemented to test as well as validate HLT. The results of the instructional video recordings were analyzed in the metapedadidactic stage. The didactic design of the two matrix multiplication material contains didactic anticipation that can be an alternative for the teacher when facing situations and responses that may occur.

Keywords: Learning Obstacles; Didactic design; Multiplication of Two Matrices

Introduction

Learning As part of the delivery of education and teaching from education to tertiary institutions will be effective if it is managed by professional education staff and teachers. In addition, to improve the quality of education, especially mathematics education, efforts are made through the provision of educational facilities and infrastructure such as textbooks, teaching aids and teaching facilities (Jamal, 2014). Besides, the implementation of the learning itself in teaching and learning activities in the classroom is equally important. Less meaningful learning can also result in students understanding mathematical concepts partially, which is not integrated between one concept and another (Dedy & Sumiaty, 2017).

Mathematics learning is basically three main components that are related to each other in a didactic triangle, namely teachers, students, and material (Kansanen, 2003; Suryadi, 2013). If
learning is only based on textual understanding, it will result in a mathematics learning process that is poor in meaning and context, as well as a result-oriented learning process that causes students to learn passively. However, it cannot be denied that naturally students may experience a situation called a learning obstacle (Brousseau, 2002). Learning Obstacle is a learning obstacle or difficulty experienced by students in the learning process (Aisah & Yulianti, 2016; Alfiyyati, 2016). The study of learning obstacles is one of the writers' backgrounds for conducting this research.

There are three elements involved in multiplication operations: Scalar, Vector and Matrix (Zang, 2016). The multiplication relationship includes operations between scalar and vector, scalar and matrix, two vectors, vector and matrix, and two matrices. Learning Obstacles that occur in the multiplication of two matrices have been described in several studies. Halid (2016), in his research shows that there are still students who do not understand the basic concept of matrix multiplication and students also experience difficulties in translation operations and the final results of matrix calculations. Zang (2016) also revealed a weak relationship between relevant knowledge and matrix multiplication completion. Then the research conducted by Lesmana, Yusmin dan Sayu (2015) showed that the mistakes made by students in matrix multiplication were misconceptions when multiplying the matrix so that it used the concept of addition, multiplying two matrices by directly multiplying the corresponding money entries and miscalculated. The matrix is one of the mathematical materials studied in class XI. One part of this material is the multiplication of two matrices.

If traced, both from previous studies and studies conducted by researchers, students generally do not experience obstacles ontogenically considering that students have understood the concepts of algebraic operations, negative and positive numbers, and the form of the matrix itself. In this case, it can be said that there is no ontogenic obstacle. Ontogenic obstacle, which is the mismatch between the learning given and the level of thinking of students, causing difficulties in the process of understanding the material (Brousseau, 2002; Rohimah, 2017).

From the results of these studies, it can be seen that the difficulty of students in the multiplication of two matrices is a lack of understanding of the multiplication concept of two matrices and the inaccurate calculation process. This can be because students still rely on memorizing formulas without understanding the concept of the material. Learning obstacles in epistemological and didactic aspects seem to be the main factor in students' obstacles to understanding matrix multiplication.

The teacher has an important role in the learning process. Each concept implanted by the teacher in learning a material has its own impact on students in the next material (Fuadiah, Suryadi & Turmudi, 2019a). Therefore, the teacher must always try to minimize the obstacles that can arise in the learning process by anticipating any responses that may occur during learning according to the learning trajectory of their students (learning trajectory). This didactic anticipation aims to make the teacher ready for any situation that develops during learning through appropriate teaching materials. Before the learning process begins, the teacher should prepare the teaching materials used and the delivery strategy so that the teaching materials can be easily conveyed to students (Sanchez & Vacarcel, 1999).

Learning trajectory (LT) is a student learning trajectory that describes the steps of student thinking in achieving learning goals (Alfiyyati, 2016). Clements dan Sarama (2007) explain that the learning trajectory is a description of students' thoughts during the learning process in the form of assumptions and hypotheses from a series of learning designs to encourage the development of student thinking so that the objectives of learning mathematics are as expected. The didactic design was originally outlined in the Hypothetical Learning Trajectory, which is a
form of learning trajectory prepared by the teacher based on the idea of choosing a specific learning design, so that the best learning outcomes are more likely to be achieved (Wijaya, 2015; Wilson, Sztajn, Edgington, & Confrey, 2013).

Teaching materials are made there must be alternative learning to overcome problems in the learning process which illustrates an effort to facilitate learning trajectories or student learning flows through a didactic situation that can encourage students to construct their knowledge. In the didactic situation created by the teacher, students are given the freedom to build up their own knowledge gradually and continuously. Brousseau (2002) identifies this didactic situation in four types of situations, namely action, formulation, validation, and institutionalization (Artigue 2014; Brousseau 2002; Wisdom 2014).

Based on the background of problems related to matrix multiplication learning, the problem formulations of this study are: 1) what are the learning obstacles associated with the multiplication of two matrices? 2) What is the didactic design in learning multiplication of two matrices?

This study focuses on a didactic design based on learning obstacles in the multiplication of two matrices in class XI SMK Madyatama Palembang. The sub-focus of this research is to analyze learning obstacles in learning mathematics on the multiplication of two matrices. The benefits that will be obtained in this study are for teachers, it can be used as an alternative to try to make didactic designs, and for schools, as a reference in the development and improvement of learning in schools.

Method

This research method is Didactical Design Research (DDR) for the material of multiplication of two matrices. DDR is based on two paradigms, namely interpretive and critical (Suryadi, 2018). Participants involved in this study were 21 students of class XI at SMK Madyatama Palembang and a mathematics teacher at SMK Madyatama Palembang to be used as a research site.

In collecting data, it is done by recording video of learning activities, primary data sources of test answers, results of learning analysis, results of interviews and results of observations in research conducted by researchers, and secondary data, data obtained from other research results, such as research journals.

There are three stages in the research using the DDR research framework, namely (a) perspective analysis (b) metapedadactic analysis and (c) retrospective analysis (Aminudin, Dirgantor, & Rusnayati, 2016; Dedy & Sumiaty, 2017; Suryadi, 2013).

1) Prospective analysis phase Prospective Analysis in the form of hypothetical didactic design including Didactic and Pedagogical Analysis (ADP). There are several steps in the prospective analysis stage research, namely:

a. Selection of a mathematical concept that will be used as material in research, and finally choosing the material of multiplication of two matrices.
b. Re-deepening and reviewing the characteristics of the two matrix multiplication material.
c. Literature study to determine the existence of a learning obstacle and the types of learning obstacle in the multiplication of two matrices.
d. Analysis of data that has been obtained from literature studies, then written in the background.
e. The making of a test question instrument learning obstacle to strengthen the visible learning obstacle in order to obtain sufficient data, this test question instrument learning obstacle is a diagnostic test for class XII at SMK Madyatama Palembang and a prerequisite material test for class XI at SMK Madyatama Palembang.

f. Validation of the test questions learning obstacle with competent experts in their fields.

g. Identification test Learning obstacle in the selected class

h. Analysis of test results and identifying learning obstacle experienced by students is seen from 3 aspects, namely: test questions learning obstacle, peer de briefing teacher lesson plans, and interviews with subject teachers. Conducting interviews with mathematics subject teachers who teach the class that will be used as research.

i. The HLT design is based on a learning obstacle and validates with competent experts on the multiplication concept of two matrices.

j. The formulation of a hypothetical didactic design, this design was made by considering the characteristics of class XI students of SMK Madyatama Palembang. In addition, this didactic design is also made based learning trajectory and learning obstacle on the student's concept of multiplication of two matrices and validates with experts who are competent in their fields regarding the concept of multiplying two matrices.

2) Metapedadidk Coalition Stage

The metapedadactic analysis stage is the analysis of the teacher's ability which includes three integrated components, namely unity, flexibility and coherence in learning. The steps in the metapedadactic analysis stage are as follows:

a. Implementation of didactic designs that have been made through trials (teaching experiment). Included in this activity is the final identification test that aims to what extent learning obstacles can be overcome.

b. Analysis of the didactic situation when the didactic design was applied from various student responses.

c. Analysis of the final identification test was performed

3) Retrospective analysis phase

Retrospective analysis stage, namely the analysis that links the results of the didactic situation analysis hypothesis with the results of the metapedadactic analysis. There are steps in the retrospective analysis stage, namely:

a. The study of the predictions of student responses and the anticipations that have been made with student responses that occur during the implementation of the didactic design.

b. Analyze and review the effect on a hypothetical didactic design using a final identification test.

c. Revise the hypothetical didactic design that has been made so that it is hoped that the new didactic design will be better than the hypothetical didactic design.

d. Prepare a research report.

The data collection used in this research is to use: 1) the observations made that are directly involved in school. Analyze the teaching materials used by the teacher in classroom learning, then analyze the Learning Implementation Plan (RPP) used by the teacher, and provide diagnostic tests for students to determine learning obstacles experienced by students. 2) The interview was conducted on the mathematics learning teacher of SMK Madyatama Palembang. By submitting the opening questions and then proceeding with questions that lead
to student difficulties in the multiplication of two matrices material and the difficulty of the
teacher in delivering mathematics learning material, the results of the interview can identify the
causes of the emergence of learning obstacle. This interview was conducted with the aim of
extracting deeper information from the research subject. 3) Documentation is done by collecting
students' work answers. 4) Video recording is used to document student activities during the
learning process both individually and in groups. 5) The test is collected from 3 types of tests,
namely: 1. The diagnostic test aims to see the learning obstacle, there are seven questions in the
form of descriptions, this test is carried out in a class that has studied before, 2. The prerequisite
material test is to see the initial conditions of students who will be participants in In this study,
there are four descriptive questions, and 3. the final identification test aims to see student
achievement after the application of a hypothetical didactic design. There are four description
questions. 6) study of the lesson plan of the teacher to see the learning activities carried out by
the teacher while teaching the multiplication of two matrices in class XI SMK Madyatama
Palembang.

The data validity techniques in this study were 1) validity including diagnostic tests,
prerequisite material tests, final identification tests, didactic design 2). Reliability includes
triangulation of checked data is the learning design and the results of the video recording of the
lesson, interviews with the teacher are given to the teacher concerned. 2) cross interpretation, in
this study the learning video recording data to be asked for opinion whether it is appropriate or
not with the learning design made. Data analysis techniques in this study include data
reduction, categorization, and synthesis.

Result and Discussion

This research consists of three stages, namely: a prospective stage, a metapedadactic stage and a
retrospective stage.

1. Prospective

Stage The prospective stage in this study conducted material analysis, identified
learning obstacles, designed the HLT, compiled a hypothetical didactic design and validated
the hypothetical design. The following is a discussion of the prospective stage of the
process.

a. Material Analysis

The material in this study is the multiplication of two matrices taught by class XI SMK
Madyatama Palembang. This school uses the 2013 curriculum. Matrix material is studied
from high school using the 2013 curriculum in class XI odd semesters.

Class XI at SMK Madyatama Palembang uses the 2013 curriculum with two matrix
multiplication materials taught in class XI semester odd with the subject matter of the
matrix. The indicators on the multiplication of two class XI matrices are as follows:

1) Determine the multiplication of two matrices
2) Calculate the multiplication of two matrices
3) Solve the multiplication of two matrices related to everyday life.

Furthermore, from the study the research curriculum examines the learning obstacle
which is a problem faced by students when learning takes place.

The identification of the learning obstacle in this study was obtained from the results of the analysis of the diagnostic test, the results of interviews with mathematics subject teachers and the results of the analysis of the teacher's lesson plans used in the learning process.

This diagnostic test aims to examine errors that occur in students in the multiplication of two matrices. This test was carried out on students who had studied the material, namely 32 students of class XII who took the diagnostic test with 7 questions in the form of descriptions. Before it was carried out on students, the diagnostic test first conducted validation on two validators, namely lecturers and teachers of mathematics subjects. Validation of this diagnostic test has improved, namely the reduction of two questions and the addition of one question, (can be seen on page 90). Furthermore, the diagnostic test can be given to students.

This design was developed based on the HLT that had been made before. After the didactic design then the didactic design that had been designed according to Basic Competencies, Indicators and Learning Objectives with several writing revisions. After being validated, then entered the Metapedadidactic Analysis stage.

2. Metapedadidactic analysis

There are several things done at the metapedadidactic analysis stage, namely the prerequisite material test to determine the student's initial condition, the hypothetical didactic design trial and the final identification test to see the student's condition after the application of the hypothetical didactic design.

The prerequisite material test aims to see the initial conditions of students who will become participants in the research. This test has 4 questions in the form of descriptions conducted in class XI SMK Madyatama Palembang, totaling 21 students, but when this test was conducted there was 1 student who did not attend school so only 20 students took the test.

The aspects seen in this study are understanding matrix order and understanding scalar multiplication. Furthermore, after the test, the researcher analyzed the results of the test. From the results of the test analysis, the following results were obtained:

a. Students have no difficulty in understanding the matrix order
b. Students have no difficulty in understanding the concept of scalar multiplication

Based on the results of the identification of learning obstacles, there are several learning obstacles experienced by students in the multiplication of two matrices which cause students to have difficulty understanding the concept of multiplication of two matrices, this is reinforced by Brousseau's opinion (Suryadi, 2016) which states that learning obstacles are obstacles or learning difficulties that faced by students in the learning process. Knowing the learning obstacle in students who have learned the multiplication of two matrices before, by doing an instrument test such as giving a learning obstacle test. This test is given in the form of description questions and there are prerequisite material questions as a basis for students to learn, before entering the multiplication of two matrices.

Based on the results of the analysis learning obstacle that emerged were: 1) a misconception of multiplying two matrices. 2) error in calculating the multiplication of two matrices. 3) errors on the daily problem of the matrix. This is in line with the research of Lesmana, Yusmin dan Sayu (2015) with the results of the study showing that some errors that occur are misconceptions when multiplying the matrix so that it uses the concept of addition, multiplying two matrices by directly multiplying corresponding money entries and miscalculating. It seems
that students use the concept of multiplication in arithmetic as the basis for the concept of matrix multiplication. This then creates an epistemological barrier, namely the existence of student misunderstandings in the concept of matrix multiplication. This epistemological constraint not only has some connection with conflict-cognitive or socio-cognitive, but also conceptions or more precisely misconceptions (Schneider, 2014).

The next stage is to examine the learning trajectory from various learning sources such as books used by teachers when teaching in class. Learning activities in textbooks are one of the causes of students experiencing learning barriers while the role of textbooks in the learning process is very important and is the main source needed for teachers and students, this is reinforced by the opinion of Fuadiah (2017) which states textbooks are one of the main sources what teachers and students need themselves to be used as material or as information in the teaching and learning process in class. Based on the teacher’s interview in mathematics, students still find it difficult to determine the multiplication of two matrices and it is still difficult to calculate the multiplication of two matrices. Because it is necessary to design a learning design to overcome learning obstacle the emerging.

Then from the learning obstacle that emerged the researcher designed the Hypothetical Learning Trajectory (HLT) which was then developed into a learning design in the form of a didactic design. HLT is structured based on learning barriers found from epistemological and didactic aspects that are in accordance with the students' mindset. HLT is designed based on learning objectives to be achieved, activities that support goals, and mathematical hypotheses in the form of conjectures that are expected to occur in students according to their thinking abilities (Clement & Sarama, 2007).

With this didactic design, the teacher can overcome didactic problems that occur in class and can create conducive learning interactions. Then also developed a learning media which later can support the teaching and learning process in the design which will make it easier for students to understand a learning concept. Based on the student responses that appear, of course there is anticipation given by the researcher to overcome problems that may occur during learning.

The didactic situation theory developed in this study is the provision of questions listed on the activity sheet and also the provision of learning media which is a tool for working on activity sheets. In line with Fuadiah (2017) the didactic situation created by teachers through didactic design in classroom learning activities is expected to develop its potential, through a series of abstract processes that can build their own knowledge. In the given activity, it begins with a problem or question that aims to enable students to use their creative thinking, for example by asking which question is the multiplication product of the matrix when viewed from the number of orders, why $A_{2\times3} B_{3\times2} = C_{2\times2}$. This is in accordance with Piaget's theory that students start the learning process when they are in an environment full of difficulties and obstacles such as those that occur in adults in general (Slavin, 2011).

When viewed from the activity sheet completing the multiplication of two matrices in everyday life and the final identification test which was done individually, students succeeded in answering these questions. when the didactic design was applied, there was a change in the didactic situation with a hypothetical design that was made because students needed more questions to make students better understand the concept or material presented and also students were less focused on learning. This action is a scaffolding, namely giving a number of assistance to a student during the early stages of learning and then reducing the assistance and giving students the opportunity to take over greater responsibility after he/she has done it (Baxter & Williams, 2010).
In teaching and learning activities in the classroom, students respond well to the development of the didactic situation, the students' responses are in line with the didactic situation developed. This is in line with Suryadi (2013) regarding student responses that have been predicted by the teacher in learning which is contained in the teacher's thinking activities before learning begins.

The learning situation occurs when a hypothetical design is applied, the learning situation actually occurs along the stages that are passed so that students respond to the didactic situation well, but there are some students who do not respond to all didactic situations because students are just following the lesson. Each student has a different response to the didactic situation that is developed, as well as the situation when starting to learn there are various forms such as laziness or playing games. Meanwhile, students learn really in order to succeed in learning and face the existing difficulties. However, these students try to solve the problem by asking the teacher or their friends so that students can communicate what is on their mind.

HLT and hypothetical didactic designs can change because after doing didactic design trials, didactic design trials can become a revision which will make the didactic design perfect for class XI SMK Madyatama Palembang.

Based on Design Trials and Metapedadidactic Analysis There are several Design Revisions, namely:

1) Determine the multiplication of two matrices

In the material determining the multiplication operation of two matrices by giving multiplication questions of two matrices with the help of learning media in the form of a matrix made of stereofomes so that it can determine the multiplication procedure of two matrices and questioning the relationship of the matrix order with the multiplication of two matrices so that students find the multiplication requirements of two matrices. The multiplication of two matrices requires that the number of columns of the first factor A must equal the number of rows of the second factor B in order to form the product \( AB \). (Anton & Rorres, 2002) Then students discuss the multiplication of the two matrices they found, then give individual practice questions to see the students' abilities in this material and students give conclusions on the material that has been presented.

2) Calculating the multiplication of two matrices

At this stage, giving questions and students looking for answers in discussion with their group friends, then discussing the multiplication of the two matrices they found followed by a presentation of the work they found. Furthermore, giving exercise questions individually to see the students' ability in calculating material and presenting in the future the results they found to see the extent of their understanding of the concept of the material. In the learning process of mathematics, mathematical understanding is a very important part (Destiniar, 2016). Students' understanding becomes the concept of matrix multiplication which is different from scalar multiplication or becomes the main goal in this learning process. Finally, students convey conclusions on the material that has been delivered today.

3) Solving contextual problems related to multiplying two matrices

Based on their research findings, Ulya, Irawati, and Maulana, Ulya, Irawati, dan Maulana (2016) suggest that the learning process using a contextual approach is able to provide opportunities for students to be able to find and construct their own knowledge in understanding learning material. Solving contextual problems related to the multiplication of two matrices is implemented by giving questions and students looking for an answer in discussion with their group of friends. Then, from the results of the students' work, they
presented what they were doing. The last stage is giving practice questions individually to see the success in this material and students give conclusions on this material. The existence of discussions between students in determining the right strategy is an activity that is useful for the development of mathematics itself (Fuadiah, Suryadi & Turmudi, 2019b). Thus it can be concluded that the didactic situation designed by the teacher made students learn something.

Conclusion

Based on the research and discussion that has been carried out, it can be concluded that the Learning obstacle that appears in the multiplication of two matrices material, namely: determining the multiplication concept of two matrices, calculating the multiplication of two matrices and solving problems related to the matrix. The initial didactic design of the multiplication concept of two matrices consists of 3 activities. The main points are: determining the multiplication of two matrices, calculating the multiplication of two matrices, and solving problems related to the multiplication of two matrices. This didactic design for the material to determine the multiplication of two matrices, the researcher uses learning media in the form of a matrix made of sterofomes.

The suggestions from the researcher are based on the results of the research, because this observation of learning obstacle didactic is only carried out through the Learning Implementation Plan (RPP) it is recommended to observe direct learning activities by taking the video recordings directly. And this research is limited to the learning media used to determine the multiplication of two matrices made of sterofom and cardboard, it is suggested that the learning media used should be made of wood or plywood so that the learning media is durable.

References


